Mathematics

Class VII

National Curriculum and Textbook Board, Bangladesh

বিজয় উল্লাস : ১৯৭১

১৯৪৭ সাল থেকেই পাকিস্তানি শাসকগোষ্ঠী দ্বারা পূর্ব পাকিস্তানের (বর্তমান বাংলাদেশ) জনগণ সর্বপ্রকার অত্যাচার, শোষণ, বৈষম্য ও নিপীড়নের শিকার হয়েছে। ১৯৭১ সালের ৭ই মার্চ বাংলাদেশের স্বাধীনতা সংগ্রামের অবিসংবাদিত নেতা বঙ্গবন্ধু শেখ মুজিবুর রহমান স্বাধীনতার ডাক দেন এবং ২৬শে মার্চ আনুষ্ঠানিকভাবে স্বাধীনতার ঘোষণা প্রদান করেন। ৯ মাসের মুক্তিযুদ্ধে অংশ নেয় নারী-পুরুষ, হিন্দু-মুসলিম, বৌদ্ধ-খ্রিষ্টান, শিশু-কিশোরসহ সর্বস্তরের জনগণ। পাকিস্তানি সেনাদের পাশবিক নির্যাতনের শিকার ২ লাখের অধিক মা-বোনের ত্যাগ এবং ৩০ লক্ষ বাঙালির প্রাণের বিনিময়ে সশস্ত্র সংগ্রামের মাধ্যমে ১৯৭১ সালে ১৬ই ডিসেম্বর মুক্তিবাহিনী ও ভারতীয় বাহিনীর যৌথ কমান্ডের কাছে পাকিস্তানি হানাদার বাহিনীর আত্মসমর্পণের মধ্য দিয়ে মুক্তিযুদ্ধে বিজয় অর্জন করে বাংলাদেশ। বিশ্ব ইতিহাসে বাংলাদেশের মুক্তিযুদ্ধ খুবই তাৎপর্যপূর্ণ ঘটনা। বাংলাদেশ তৃতীয় বিশ্বের প্রথম দেশ, যে দেশ সশস্ত্র মুক্তিযুদ্ধের মাধ্যমে স্বাধীনতা অর্জন করেছে।

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Mathematics

Class VII

(Experimental version)

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Preface

In this ever-changing world, the concept of livelihood is altering every moment. The advancement of technology, in accordance with knowledge and skill, has accelerated the pace of change. There is no alternative to adapting to this fast changing world. The reason is, the development of technology is at its zenith compared to any time in the human history. In the fourth industrial revolution era, the advancement of artificial intelligence has brought a drastic change in our employment and lifestyles and this will make the relationship among people more and more intimate. Varied employment opportunities will be created in near future which we cannot even predict at this moment. We need to take preparation right now so that we can adapt ourselves to that upcoming future.

Although a huge economic development has taken place throughout the world, the problems of climate change, air pollution, migrations and ethnic violence have become much more intense than before. The epidemics like COVID 19 has appeared and obstructed the normal lifestyle and economic growth of the world. Different challenges and opportunities have been added to our daily life.

Standing on the verge of these challenges and possibilities, implementation of sustainable and effective solutions is required for the transformation of our large population into a resource. It entails global citizens with knowledge, skill, values, vision, positive attitude, sensitivity, capability to adapt, humanity and patriotism. Amidst all these, Bangladesh has graduated into a developing nation from the underdeveloped periphery and is continuously trying to achieve the desired goals in order to become a developed country by 2041. Education is one of the pivotal instruments to attain the goals and there is no alternative to the modernization of our education system. Developing an effective and updated curriculum has become crucial for this modernization.

Developing and revising the curriculum is a regular and vital activity of National Curriculum and Textbook Board. The last revision of the curriculum was done in 2012. Since then, a lot of time has passed. The necessity of curriculum revision and development has emerged. For this purpose, various research and technical exercises were conducted under the supervision of NCTB during the year 2017 to 2019 to analyze the prevalent situation of education and assess the learning needs. Based on the researches and technical exercises, a competency-based incessant curriculum from K-12 has been developed to create a competent generation to survive in the new world situation.

In the light of the competency based curriculum, the textbooks have been prepared for all streams (General, Madrasah and Vocational) of learners for grade VII. The authentic experience driven contents of this textbook were developed in such a way that teaching learning becomes comprehensible and full of merriment. This will connect textbooks with various life related phenomenon and events that are constantly taking place around us. We hope that learning will be profound and life-long now.

Issues like gender, ethnicity, religion, caste, the disadvantaged and students with special needs have been taken into special consideration while developing the textbook. I would like to thank all who have put their best efforts in writing, editing, illustrating and publishing the textbook.

If any one finds any errors or inconsistencies in this experimental version and has any suggestions for improving its quality, we kindly ask them to let us know.

Professor Md. Farhadul Islam
Chairman
National Curriculum and Textbook Board, Bangladesh

Dear Student,

Welcome to the world of Mathematics of grade seven.

You know that there have been several changes in the Mathematics book of class seven. The way you have been learning mathematics in various chapters till now has changed a lot. Throughout the year, you will go through several learning experiences. In different stages of these experiences, you will need to know about various aspects of mathematics, for which you can take the help of this book.

Each experience is designed in such a way so that you can master various mathematical skills through problem solving and become adept at solving real-life problems. You will participate in a variety of experiences through group, pair or individual work inside and outside the classroom. Your teacher will support you throughout this journey. This textbook will act as a helpful tool for you at various stages of learning. We hope that by working on the use of mathematics, you will realize the importance of mathematics in real life and be more interested in learning mathematics.

Best wishes to you all.

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Exponent

Multiplication Game

Let's read a story.

There was a king in a region. There was no match for that king in wealth and prestige. The king was very proud of this wealth. Once a strange traveler came to that king and brought a very nice painting. The king wanted to pay the price of the painting, but the traveler asked for that in a strange way. He wanted the price to be paid for 50 consecutive days. On the very first day he used to ask for one taka. On the second day, he took twice that; 2 taka. Again, the next day he used to take twice of the second day; that is, 4 taka. In the same way, he took the price for 50 days. It was like the following table.

Table 0.1

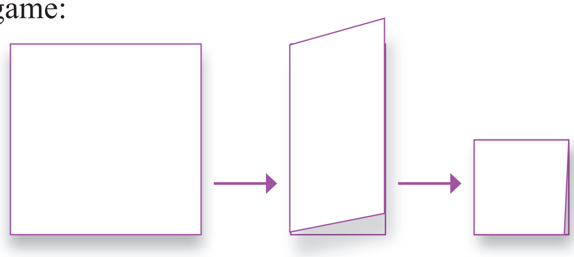
Day	Multiplication	Amount of money
1		1
2	12	2
3	22	4
4	42	8
… … … …		

Thus he took money every day. However after 20 days, the king and his minister realized the matter and the king apologized to the traveler for his arrogance. Think, what was the reason for this? Make a table in your notebook like table 0.1 and write the amount of money paid from day 5 to day 20.

Did you understand how did the traveler establish this way of taking money? Let's play a game to understand this more.

The game of paper folding

We will follow the steps below for this game:



1. Each of you is required to take a sheet of A4 paper or a paper from a big notebook.
2. Draw lines around the paper with a pen. (Draw the lines along the four sides in such a way that each of the drawn line seems to you like a margin).
3. Now fold the paper into two equal parts. Then open the fold and draw line along the folding mark. Then how many cells are there with margins on four sides? Certainly 2.
4. Again, fold the paper into two equal parts in this folding state and draw lines accordingly. Now how many total cells are there?
5. Now do the task three more times and fill in the following table (1.1) calculating the number of cells.

After that, instead of twofold make a threefold each time and fold a total of 4 times. Fill in the following table (1.2) like the table (1.1).

Table 1.1

Folding number	Number of cells
1	2
2	
3	
4	
5	

Table 1.2

Folding number	Number of cells
1	3
2	
3	
4	

Now, let's do a task sitting in the classroom. Those of you have even roll numbers will write the number 6 in the following table. Those with odd roll numbers will write the number 5 in the following table.

Table 1.3

Number	How many numbers are there?
□	

Now, multiply the number you have taken with the same number once and fill in like the following table. Think, what will happen? If your roll is odd, then 5 will be multiplication form in two times. That is, multiplication form will be 55. If your roll is even, then 6 will be multiplication form in two times. That is, multiplication form will be 66.

Table 1.4

Multiplication form	Product	How many numbers are used in Multiplication form?
□×□		

Now in the same way, multiply 2 times with that number and write as multiplication form in the following table. What is the product?

Table 1.5

Multiplication form	Product	How many numbers are used in Multiplication form?
□×□×□		

In a similar way, multiply 3 times, 4 times and 5 times and write in the following table. For convenience, the table is partially filled in.

Table 1.6

Multiplication form	Product	How many numbers are used in Multiplication form?
□×□×□×□		

After completing the table, do another task. Now multiply the number 10 times, 11 times and 12 times and write only as multiplication form in the following table.

Table 1.7

Multiplication form	How many numbers are used in Multiplication form?

You can notice that it takes a large amount of space and time to write the multiplication form. However, it is possible to write these large multiplication forms in a very easy way in small space and in short time.

Think about every case in the table 1.3 – 1.6, how many numbers were there in multiplication form in each case? We can write the multiplication form very easily in another way. Notice the table 1.8.

Table 1.8

Multiplication form	product	How many numbers are used in Multiplication form?	New way of product writing
1010	100	2	
101010	1000	3	
10101010	10000	4	
1010101010	100000	5	

Do you understand what is happening here? Here the number written in multiplication form is written first and then the number of times of that number is written at the top right side of it. Now see if you can do it yourself. Fill in the following table.

Table 1.9

Which was the number taken by you? 5 or 6?	Multiplication form	Product	How many numbers are used in Multiplication form?	New way of product writing
			2	$\square^2$
			3	$\square^3$
			4	$\square^4$
			5	$\square^5$
			6	$\square^6$

Now think. You filled in the table (1.7) multiplying 10 times, 11 times and 12 times with the number taken by you. It was hard to do the task, wasn't it? So, see whether you can write in the following table according to the new method learnt by you.

Table 1.10

Which was the number taken by you? 5 or 6?	Multiplication form	Product	How many numbers are used in Multiplication form?	New way of product writing

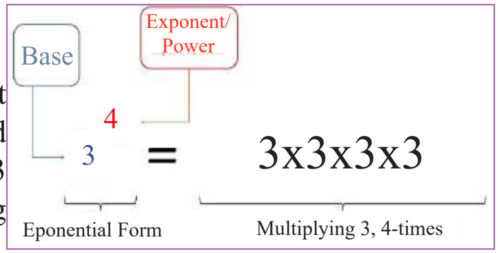


figure 7.2.3

Notice the figure 7.2.3, we write the product of 3 with itself 4 times. Here we have placed a small sized 4 in the upper right corner of 3 instead writing the number repeatedly using multiplication sign.

The number, putting above the number you have taken, is called exponent or power. The number we are multiplying repeatedly is called base. Here 3 is the base and 4 is the exponent or power.

So we can understand that using exponents we can easily express large multiplications in a shorter way. Let's see how we will read the number when we are using exponents.

Exponential form	How to read
3^2	3 to the power 2 [We can also say 3 squared]
3^3	3 to the power 3 [We can also say 3 cubed]
3^4	3 to the power 4
3^5	3 to the power 5

This way of writing large multiplications in a shorter way is called exponential system.

Now let's think about another thing. We have seen in multiplication form that we can consider the number of times the fixed number or base is counted as exponent or power. If you can't understand, see the above figure (7.2.3) again.

Now let's see an example from the table 1.8

$10^3=10\times10\times10$

Here three 10s are in a multiplication form. So 3 is as the power of 10. So think, what did you do in the table 1.3? Count, how many numbers were there? However, there was only one number. Again, we can say as an example, if we write only 10, there is only one 10. In this case, it can be expressed as exponential also. And the power or exponent will be according to the new rule we learnt. That is, only one number or 10 is written as .

So, fill up the table 1.11 is which is partially done for your help. Draw a similar table in your notebook and fill that up using number 9 as the base.

Table 1.11

Number	Power	Write in Multiplication form	Write in exponential method	Product
10	1	10	10^1	10
	2	10×10		100
	3		10^2	1000
	4	10×10×10×10		10000
	5		15^5	100000
	6	10×10×10×10×10×10		1000000

We hope, in the meantime, you have gotten elaborate concept of exponent. Now let's try to fill up the following table.

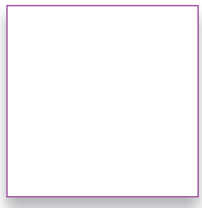
Table 1.12

Multiplication form	Exponential form	Base	Power
7×7×7×7×7×7×7×7×7×7×7×7×7×7×7			
4×4×4×4×4×4×4			
14×14×14×14×14			
2×2×2×2×2×2×2×2×2×2			
8×8×8			
11×11×11×11×11×11×11×11			
21			

Let's think about the game of folding paper again. Can you find the concept of exponent in that game? If you can, then fill up the following table 1.13 and make a similar table in your notebook to fill up for threefold.

Table 1.13

Nature of folding	Number of folding	Number of cells	Exponential form	Multiplication form
Folding into two equal parts each time	1	2		
	2			
	3			
	4			
	5			



Now think, when you did not fold the paper, drawn line along the four sides on the whole paper formed a cell.

So, when you did not fold, the number of folding is 0. But how many cells are there? There is one cell. Now see another interesting thing. No matter how many positive numbers you want to fold each time, the first time that is at zero folding, there is only one cell. Can you understand something from this?

Task:

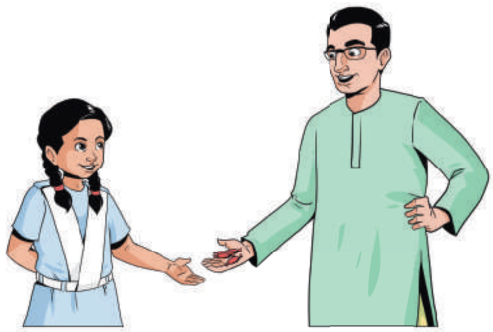
1) Complete the table of problems of the proud king in your notebook like the table below.

Day	Exponential form	Amount of money
1		1
2	2^1	2
	…………………	
29		
30		

Exponent of 0 and 1

Your school authority has decided, Candy will be given in your class for a total of five days. But there are some rules in that case.

Firstly who will get how many candies will depend on each person's roll number. These candies will be given with respect to the last digit of each student's roll number. However, those who have one digit roll that one digit will be the acceptable number.



Now how the last digit of roll no. will determine the number of candies?

First day, the number of candies will be given to a student as per the last digit of the roll.

Next day, that is 2nd day, the number of candies of a student will be the number of candies of the previous day multiplied by the last digit of roll.

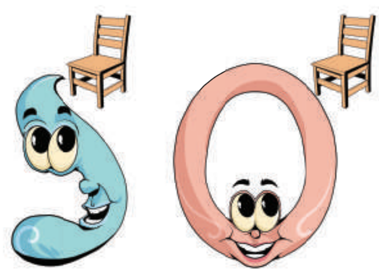
The 3rd day, the number of candies of a student will be the number of candies of the previous day (2nd day) multiplied by the last digit of roll.

According to this rule, everyone will get candies for the remaining two days.

At first think about your roll no. and take the last digit of your roll. According to the rule, if your roll is one digit number, then that number will be the last digit of your roll or acceptable number. Then, fill in the following table.

Table 1.18

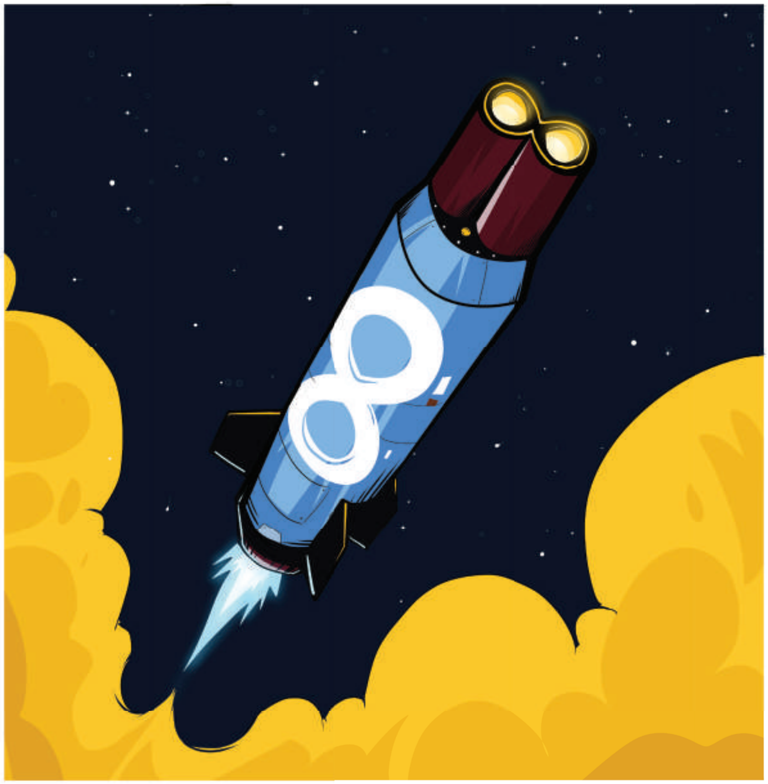
Roll	Last digit of roll	day	The number of received candies
□	□	1st day	□
		2nd day	□×□
		3rd day	□×□×□
		4th day	
		5th day	



Now see the matter. Those who had 0 or 1 at the end of the roll of your class, how many candies did they get after five days? Or, how many candies did they get every day?

If you notice carefully, those who have last digit 0 in their roll, they will get no candy. Again, those who have last digit 1in his roll, they will get one candy every day. That is, they have no change in the number of candy. That is, if you put exponent upon 0 or 1, they will be 0 or 1 respectively. But remember, 0 can't be exponent of 0. Can you think why is that?

Work with Exponent:



We learn the story of a strange spaceship now. Why are we using the term 'strange'? It's because the velocity of this spaceship is always based on 4. That is, its velocity is some positive power of 4 per second. To make it simpler, the distance traveled by the spaceship in one second will be some positive power of 4. As an example, we can think of . That is, the spaceship will travel a distance of metres in one second. However, it should be remembered that this velocity is not fixed. It may increase or decrease. It is only sure that the velocity will always be a power of 4.

The pilot of the spaceship can see in the monitor of the spaceship, how far the spaceship did travel with respect to time. But the interesting thing is, the time is shown as power of 4 in that monitor. That is, pilot can't see the distance after two seconds as he wish. He can see the distance traveled by this spaceship such as at time interval seconds or seconds. The time in the monitor will maintain a sequence. As for example, at first pilot can see the distance traveled in time interval seconds. Next, after seconds, he can see the distance traveled by next seconds. Again after seconds, he can see the distance traveled by next seconds and this will continue. Remember that it is not possible to see the distance traveled by next seconds after seconds.

One day while piloting the spaceship, the pilot noticed that the velocity is fixed and the velocity is metres per seconed. It is not increasing or not decreasing. First he saw the distance traveled after second. After seeing the distance traveled in the next second, the spaceship jolted suddenly and no elapsed distance was shown on the monitor from the next time interval. The pilot of the spaceship got into trouble. It was important to know the elapsed distances. Can you help the pilot of the spaceship?

Now think, the spaceship traveled metre in one second. So how much distance will it travel in second? We can easily see from the concept of unitary rules, the distance traveled by the spaceship in 4^1 seconds is

So, can you find the distance traveled by the spaceship in the 2nd time interval?

The spaceship traveled metre in one second.

The distance traveled by the spaceship in second is metre.

Table 2.1 (Filled up partially. If needed draw the table in your notebook and complete it)

Time interval (second)	Speed (metre/ second)	Multiple form of elapsed distance (metre)	Elapsed distance (Exponential form) (metre)
4^1	4	$4^1\times4=4\times4$	4^2
4^2	4	$4^2\times4=4\times4\times4$	4^3
4^3	4		
4^4	4		
4^5	4		
4^6	4		
4^7	4		

In this way, after passing the above seven time intervals, the pilot landed the spaceship and instructed the technical team to fix the monitor error. However the next day the pilot had to fly the spaceship again for an urgent reason. As a result, the monitor error remains. However, while the previous day the pilot could see the distance traveled in the first two time intervals, this day he could only see the distance traveled in the first time interval and he could not see the distance traveled during the rest of time intervals. There is another difference this day. The previous day, the velocity of the spaceship was the same at each time interval, but this day its velocity is different at each time interval. The time interval and speed of his spaceship are given in the table. Can you help the pilot determine the elapsed distance at every time interval?

Table 2.2

Time interval (second)	Speed (metre/ second)	Multiple form of elapsed distance (metre)	Elapsed distance (Exponential form) (metre)
4^1	4^5	$4^1 \times 4^5 = (4) \times (4\times4\times4\times4\times4) = 4\times4\times4\times4\times4\times4$	4^6
4^2	4^8		
4^3	4^3		
4^4	4^{10}		
4^5	4^4		
4^6	4^2		
4^7	4^9		
4^8	4		

Now what do you have to do to find the distance traveled in each given time interval? Each time you have to write in multiple form breaking the exponent form. Then you have to count the total of multiple form numbers. Then it has to be written again in exponential form. It must take a lot of time and a lot of hard work. However, we have seen, large number of multiplication can be written with the help of exponent easily and in less time. But if we have to work with large number of multiplication form, does the work become easier? So let's learn another new thing. Also in this case, I will take the help of your even and odd rolls. That is, even rolls will use the number 6 and odd rolls will use the number 5.

Observe the following table 2.3 carefully. The whole table has been filled up for your help. Next time you have to fill up the table 2.4 with the help of this table.

Table 2.3

(In the table, 10 is considered as the base of multiplication.)

Base number	**Multiplication**	**1st term of multiplication**	**Multiplication form of first term**	**2nd term of multiplication**	**Multiplication form of 2nd term**	**product**	**Exponential form of product**
	$10^2 \times 10^2$	10^2	10×10	10^4	$10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6
	$10^3 \times 10^3$	10^3	$10 \times 10 \times 10$	10^3	$10 \times 10 \times 10$	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6
10	$10^4 \times 10^1$	10^4	$10 \times 10 \times 10 \times 10$	10^1	10	$10 \times 10 \times 10 \times 10 \times 10$	10^5
	$10^2 \times 10^1$	10^2	10×10	10^1	10	$10 \times 10 \times 10$	10^3
	$10^1 \times 10^3$	10^1	10	10^3	$10 \times 10 \times 10$	$10 \times 10 \times 10 \times 10$	10^4

Table 2.4

(Find the product in the following table using the base taken by you instead of 10. Do it according to the figure 2.3 and fill up the table.)

Base number	Multiplication	1st term of multiplication	Multiplication form of first term	2nd term of multiplication	Multiplication form of 2nd term	product	Exponential form of product
	$\square^2 \times \square^4$						
	$\square^1 \times \square^4$						
$\square$	$\square^3 \times \square^1$						
	$\square^2 \times \square^1$						
	$\square^3 \times \square^3$						
	$\square^3 \times \square^2$						

Now compare between the table 2.3 and table 2.4. What do you understand? When two exponential forms with same base are multiplied, the product is an exponential form with the same base. The exponent or power of new exponential form is the sum of the exponent or power of the multiple and multiplier. Then, it will be clearer with the help of the given table. The table has been partially filled up.

Table 2.5

(Fill up the table according to the serial of table 2.3 and table 2.4). The table has been partially filled up. Complete the table with your learning and with the information obtained from the two tables.

Serial No	Data obtained from table 2.3			Data obtained from table 2.4		
	Multiplication form	Step of multiplication	Product	Multiplication form	Step of multiplication	Product
1	$10^2\times10^4$	10^{2+4}	10^6	$\square^2\times\square^4$		
2	$10^3\times10^3$		10^6	$\square^1\times\square^4$		
3	$10^2\times10^3$	10^{2+3}	10	$\square^3\times\square^1$		
4	$10^4\times10^1$		10^5	$\square^2\times\square^1$		
5	$10^2\times10^1$	10^{2+1}	10	$\square^3\times\square^3$		
6	$10^1\times10^3$		10^4	$\square^3\times\square^2$		

The product of two or more exponential expressions of the same base is possible to express another exponential form of that base. The exponent of the product will be the sum of all exponents of all expressions in that multiplication form.

Task:

1. Determine the product with the help of the rule of multiplication of exponent. (If the product is 0 or 1, the base will be 0 or 1. Product will be written as you have learnt about the value of exponent.)

Serial No.	Multiplication of exponent	Product(Exponential form)
1	$7^4\times7^7$	
2	$0^8\times0^2$	
3	$1^{24}\times1^{18}$	
4	$12^{12}\times12^{12}$	
5	$71^{28}\times71^{72}$	
6	$21^{21}\times21^{14}\times21^5\times21^2$	

2) Draw a table in your exercise book like the table 2.2 and fill in according to the rule of multiplication of exponent.

3) Hasan is in a trouble in multiplying numbers of two exponential form. These two numbers are and. He writes the two numbers two times in multiplication form like

the table. Is he writing correctly?

$5^2 \times 12^2 = 5^{2+2} = 5^4 = 625$	$12^2 \times 5^2 = 12^{2+2} = 12^4 = 20736$

If any of the two multiplication methods done by Hasan is correct, then you determine the product of andusing that method. If the method of multiplication done by Hasan is wrong, then identifying the error of Hasan, you determine the correct product and then determine the product of and .

Division of exponent-1

Let's try to think like the previous story of that king. But vice versa. We will think about this story dividing us into two groups. One group is named 'A' and another group is named 'B'.

Group 'A' has candies. But group 'B' has no candy. Now group 'A' will give candies to group 'B'. However, there is a rule.

The rule is, each day group 'A' will give half the number of candies given on the previous day to group 'B'. That is, next day group 'A' will give the number of candies dividing the number they have given today by 2. Remember that only integer number of candies can be given. You cannot break a candy into half or a quarter. This will continue as long as the candies can be distributed.

Suppose, group 'A' has given candies on first day. Then how many candies group 'A' will give the next day? Or the day after next day? Now fill up the table to find out that information.

Table 3.1

(If it is not possible to give candies or it is not possible to express exponential form on any day, then you will put cross (x) in that box.)

Day	Exponential form of the number of candies	Multiple form of the number of given candies
1st	2^5	$2 \times 2 \times 2 \times 2 \times 2$
2nd		$\frac{2 \times 2 \times 2 \times 2 \times 2}{2} = 2 \times 2 \times 2 \times 2 \times 2$
3rd		
4th		
5th		

6^{th}		
7^{th}		

In this way, you can count the number of candies given in the next day by knowing the number of candies given in the previous day using table. However if you were asked directly, how many candies were given on the fourth day? How would you say? Surely using table or using data of given candies everyday.

Now you imagine, at first there were candies with the group 'A'. First day they gave candies to the group 'B'. Candies are then given as usual as long as possible. Now think, if you are asked how many candies group 'B' has got on the eighth day? Determine it using the following table.

Table 3.2

Day	Exponential form of the number of candies	Multiple form of the number of given candies
1^{st}		$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
2^{nd}		$\dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2}$ $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
3^{rd}		
4^{th}		
5^{th}		
6^{th}		
7^{th}		
8^{th}		

See, this task takes a lot of effort and time also. So let's see the easy way of division of exponent like we did with multiplication.

Here we will follow a similar method like that of multiplication of exponent in the table previously. Again, you are divided into two groups basing on even or odd rolls. Even roll holders of you take the number 6 and odd roll holders of you take the number 5.

Now observe the next table 3.3 carefully. The table has been filled up completely for your help. You have to fill up the next table (Table 3.4) with the help of this.

Table 3.3

Accepted number	Division	Dividend	Multiplication form of 1st term	Divisor	Multiplication form of 2nd term	Quotient structure	Quotient	Exponential form of quotient
10	$10^4 \div 10^2$	10^4	$10\times 10 \times 10 \times 10$	10^2	10×10	$\frac{10 \times 10 \times 10 \times 10}{10 \times 10}$	10×10	10^2
	$10^3 \div 10^2$	10^3	$10 \times 10 \times 10$	10^2	10×10	$\frac{10 \times 10 \times 10}{10 \times 10}$	10	10^1
	$10^4 \div 10^1$	10^4	$10 \times 10 \times 10 \times 10$	10^1	10	$\frac{10 \times 10 \times 10 \times 10}{10}$	$10 \times 10 \times 10$	10^3
	$10^2 \div 10^1$	10^2	10×10	10^1	10	$\frac{10 \times 10}{10}$	10	10^1
	$10^1 \div 10^1$	10^1	10	10^1	10	$\frac{10}{10}$	1	?

Table 3.4:

Find the product in the following table using the base taken by you instead of 10 according to the table 3.3 and complete the table.

Accepted number	Division	Dividend	Multiplication form of 1st term	Divisor	Multiplication form of 2nd term	Quotient structure	Quotient	Exponential form of quotient
$\square$	$\square^4 \div \square^2$							
	$\square^3 \div \square^2$							
	$\square^4 \div \square^3$							
	$\square^4 \div \square^1$							

Try to compare between the table 3.3 and 3.4. What do you understand?

When two exponential forms with same base are divided, the quotient is a new exponential form with the same base. The exponent or power of new exponential form is the difference of the exponent or power of the dividend and divisor. It will be clearer with the help of the given table. The table has been partially filled up.

Table 3.5:

You have to fill up the table according to the used data in the table 3.3 and table 3.4.

The table has been partially filled up. Complete the table from your learnings and using information from the two tables.

Serial No	Data obtained from table 3.3			Data obtained from table 3.4		
	Division form	Step of division	Quent	Division form	Step of division	Quo-tient
1	$10^4 \div 10^2$	10^{4-2}	10^2	$\square^4 \div \square^2$		
2	$10^3 \div 10^2$		10^1	$\square^3 \div \square^2$		
3	$10^4 \div 10^3$			$\square^4 \div \square^3$		
4	$10^4 \div 10^1$	10^{4-3}	10^3	$\square^4 \div \square^1$		

The quotient of two exponential expressions of the same base is possible to be expressed as another exponential form of that base. In that case, the exponent of quotient will be the difference between the exponents of divisible and divisor.

When power is 0

Now observe one thing. Think about the task we have done in the last row of the table 3.3. Actually we have divided 10 by 10. But in exponential division it is $\frac{10}{10}$. Now see, what have we learnt about the division rule of exponent?

However, from that rule we can write, $\frac{10}{10} = 10^{1-1} = 10^0 = 1$

Try to remember, we played a game of folding paper at first. What did we see there? When there is no folding, then there is also a cell. That is, we got 1 cell for 0 folding. Again from the above, what have we seen using exponential rule? If the exponent is 0 upon the base 10, then it is 1. Now quickly fill up the following table.

Table 3.5 (Partially filled up)

Division	Exponential process of quotient using formula	Quotient structure	quotient	Exponential form of obtained quotient using formula
			1	

Actually, what have you seen from this? Explicitly we can say that if we divide a number by that number using simple division rule we get 0 as quotient. Now think, when exponent is 0 upon a number? When a number is divided by that number or exponential form of a number is divided by that exponential form of number. That is, if exponent is 0 upon any number then resultant value will be 1. Now, can you find any similarity with your paper folding?

Now let's think another thing. Can exponent be 0 upon 0? Here we will take help of the table 3.5. Let's think about instead of in the first row of the table. Now,

Table 3.6

Division	Exponential process of quotient using formula	Quotient structure	quotient	Exponential form of obtained quotient using formula
$0^4 \div 0^4$	0^{4-4}	$\frac{0^4}{0^4} = \square$	?	0^0

Now tell, why is it not possible? Because, we learnt that is actually 0. So, we get this quotient is . Now, is it possible to divide 0 by 0? You have seen in the class 6 that it is not possible to divide any number by 0.

So is also not possible. So, 0 can't be exponent upon 0. Think in this way, we need to divide 0 by 0 to determine in any case. We can't do that. For this, if 0 is exponent upon 0, there is no value of that exponent. Here you can also think about the folding of paper. Can you really do assuming base 0; that is, can you anyway fold a paper 0 time all the while?

If the exponent or power of any number excluding 0 is 0, the value of the exponent is 1

Division of exponent-2

Again, let's do some work using paper. We'll make a circle cutting a paper. Now cut that circle into two equal parts. So it is divided into two parts. What fraction of the circle is each portion? See that in the following table.

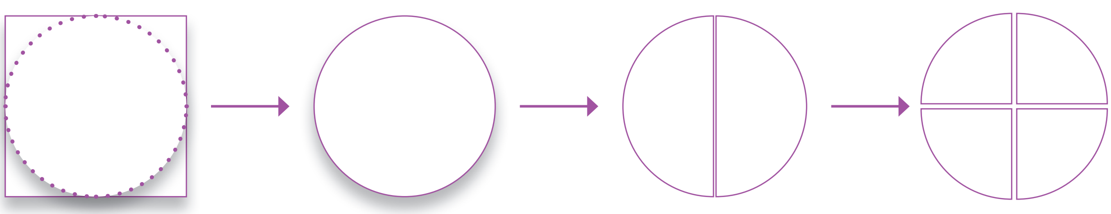


Table 4.1

Number of cutting	Number of portion	What fraction is any one portion
1	2	$\frac{1}{2}$

Cut the two portions again into two equal parts like previous and think, what fraction of the full circle is any one portion? Fill in the following table as before.

Table 4.2

Number of cutting	Number of portion	What fraction is any one portion
2		

In this way, try to do the work three times more and place the obtained data in the following table.

Table 4.3

Number of cutting	Number of portion	What fraction is any one portion
3		
4		
5		

You see that we are making pieces every time. That is, if you think generally, we have tried division here. Can you imagine the concept of exponent here? You can think about the previous idea of the division of exponent. Let's explain table 4.1 for your help. See, how many parts will we get cutting one time? It is 2 and each of the part is of the circle. Now see, we have divided the circle into two parts at each time. If you understand the game of paper folding at first, then you can tell that our base is 2. But here we have divided and we have divided systematically cutting into parts.

If you see the remaining table and if you can use the concept of exponent from there, you can understand that exponent is used here. On the other hand, when we cut into pieces, we can think it as subtraction or minus. So think, whether you can understand anything or not?

Now, let's try to continue the game that we have played between the two groups of giving candies to understand the division of exponent. The rule of the game will be the same but only one change is there. Two groups could give and take only integer number of candies in that game. However now the two groups can give and take not only integer number of candies but also fractional number of candies. That is, the portions of a candy can be divided by 2 or divided by 4 and given to the other group.

Now think, what will happen? Imagine the previous table and try to fill up the table.

Table 4.4:

If the number of candies is not possible to express in exponential form, then give cross (x) in that cell.

Day	Exponential form of given number of candies	Multiple form of given number of candies
1st		
2nd		
3rd		
4th		
5th		
6th		
7th		
8th		

Now think, what has changed really?

See, the power was not negative or zero in any case that we have shown in the meantime. Now let's see that matter. Remember that the rule of division that we have learnt in the previous is always applicable.

Let's try to learn the matter exactly like the table 3.3, but vice-versa. Here we will assume the dividend of that table as divisor and likewise, the divisor of that table becomes dividend. However we will not follow the serial like the table 3.3. Now let's see the following table.

Table 4.5

Accepted number	Division	Process of division	quotient	Quotient structure	quotient	Exponential and numerator denominator structure
10	$10^2 \div 10^3$	10^{2-3}	10^{-1}	$\frac{(10\times10)}{10\times10\times10}$	$\frac{1}{10}$	$\frac{1}{10}$
	$10^3 \div 10^4$	10^{3-4}	10^{-1}	$\frac{10\times10\times10}{10\times10\times10\times10}$	$\frac{1}{10}$	$\frac{1}{10}$
	$10^0 \div 10^1$	10^{0-1}	10^{-1}	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
	$10^2 \div 10^4$	10^{2-4}	10^{-2}	$\frac{10\times10}{10\times10\times10\times10}$	$\frac{1}{10\times10}$	$\frac{1}{10^2}$
	$10^0 \div 10^2$	10^{0-2}	10^{-2}	$\frac{1}{10\times10}$	$\frac{1}{10\times10}$	$\frac{1}{10^2}$
	$10^1 \div 10^4$	10^{1-4}	10^{-3}	$\frac{10}{10\times10\times10\times10}$	$\frac{10}{10\times10\times10}$	$\frac{1}{10^3}$

Now with the help of this fill up the table like the previous table 4.6. (Again accepted number will be 6 and 5 accordingly with your roll numbers.)

Table 4.6

Accepted number	Division	Process of division	quotient	Quotient structure	Quotient	Exponential and numerator denominator structure
□	$\square^2 \div \square^3$					
	$\square^3 \div \square^4$					
	$\square^0 \div \square^1$					
	$\square^2 \div \square^4$					
	$\square^0 \div \square^2$					
	$\square^1 \div \square^4$					

Task: 1

Serial no.	Division of exponent	Quotient	Exponential and numerator denominator structure of quotient (if applicable)
1	$11^{14} \div 11^{7}$		
2	$6^{7} \div 6^{9}$		
3	$17^{9} \div 17^{0}$		
4	$71^{71} \div 71^{8}$		
5	$19^{0} \div 19^{9}$		
6	$14^{3} \div 14^{3}$		

2) Draw table in your exercise book like the table 3.1 and the table 4.4 using the concept of division of exponent and complete the table.

3) Akash could not divide two exponential numbers at the time of dividing them. Those two numbers are and . He determined the quotient dividing two times like the table below. See whether he wrote correctly or not?

$18^{3} \div 6^{2} = 18^{3-2} = 18^{1} = 18$	$6^{2} \div 18^{3} = 6^{2-3} = 6^{-1} = \frac{1}{6}$

If any of the two methods followed by Akash is correct, then determine the quotient of and yourself.

If the methods followed by Akash are wrong, then identifying the error you determine the correct quotient. Follow the correct method and determine the quotient of and correctly.

Exponent of exponent:

Let's go back again to distribution of candy for 5 days from the school. The number of candies will be equal to the last digit of their own roll. Suppose, your school has decided that no one will be without candies this time as happened in the previous time. That was wrong. So the school authority has decided to follow a new rule to distribute candy to all for 5 days.

So think about your roll no. again and take the last digit of roll. However there is a new rule here. Since previously, students with 0 as the last digit of their roll or 1 could not get any candy at all or a few candies, so now 11 will be taken instead of the last digit of roll of those students. That is, students with 0 or 1 in the last digit of their roll will take 11 in place of that digit. Another rule has been changed. In the previous rule, a student was given candies according to the last digit of his/her roll on the first day. However now everybody will get 1 candy on the first day. Rest of the rules are same as before. 2^{nd} day, the number of candies of a student will be the number of candies of the previous day multiplied by the last digit of his/her roll. In this way everyone will get candy for the remaining three days.

Table 5.1

Roll no.	Last digit of roll	Day	Number of obtained candy
		1st	1
		2^{nd}	$1\times\square$
		3^{rd}	$1\times\square\times\square$
$\square$	$\square$	4^{th}	
		5^{th}	

Once the above table is filled up, then fill up the table below. But here you have to do group work. Group comprises of students with same last digit of the roll numbers. You have to do multiplication of the candy of you after making group. How about multiplication? The multiplication will be equal to the product of the daily number of candies you have. As for example, you have to multiply the candy each of you had on the second day. Then you have to multiply the candy each of your group had on the third day. Fill up the table in this way.

Here think about one thing before filling up the table. Let, a group gets 10 candy. And 5 members are there. So each member of the group will get 10 candies on the second day and will get on the third day. Fill up the table now.

Table 5.2

Roll no.	Last digit of roll	day	Number of candy one gets	Multiplication form of number of candy one gets	Multiplication form of number of candy of all the members of a group	Product in exponential form
		1^{st}	1	1		
		2^{nd}				
		3^{rd}				
		4^{th}				

		5^{th}				

Observe the following table after completing the above table and think what is really happening. Here we assume that we get multiple of 10 and there are 5 members of a group.

Table 5.3

One cell has been filled. Fill up rest of the cells taking help from the previous table 5.1. Similarly fill up the blank cells or partially blank cells.

Day	Number of candy one gets	Multiple form of number of candy one gets	Multiple form of number of candy getting by all the members of a group	Product in exponential form using the rule of product of exponent.
1^{st}	10^0	1	$1\times1\times1\times1\times1$	$1=10^0$
2^{nd}	10	10	$10\times10\times10\times10\times10$	10^5
3^{rd}	10^2	10×10	$10\times10\times10\times10\times10$	10^{10}
4^{th}	10^3		$10^2\times10^2\times10^2\times10^2\times10^2\times10^2$	
5^{th}	10^4		$=10^{2+2+2+2+2}$	

Think about one thing after completing the above table. We have learnt that we can write a number as exponent of a number as the times the number comes in multiplication form. Think, what do we get in the second row of the above table 5.3? There 10 appears 5 times in the multiplication form. So using the concept of exponent we get,. Now, what is there in the third row? appears 5 times in the multiplication form. So think, what has happened when we have used only 10 instead of in the previous row. Since 10 appears 5 times, so, . Likewise, we can tell also from the concept of exponent that if is there 5 times in multiplication form, then we can write . Now think, whether we can fill up the following table using concept of exponent or not?

Table 5.4

Multiplication form	Exponential form
$10\times10\times10\times10\times10$	
$10^2\times10^2\times10^2\times10^2\times10^2\times10^2$	
$14\times14\times14\times14\times14\times14\times14$	
$14^3\times14^3\times14^3\times14^3\times14^3\times14^3\times14^3$	

Now apply your learning so far to fill up the following two tables.

Table 5.5: It has been filled partially. Fill up the blank cells or partially blank cells accordingly.

Day	Number of candy one gets	Multiplication form of number of candy one gets	Multiplication form of number of candy getting by all the members of a group	Product in exponential form of exponent.
1^{st}			$1\times1\times1\times1\times1$	$(10^0)^5$
2^{nd}			$10\times10\times10\times10\times10$	$(10^1)^5$
3^{rd}			$10^2\times10^2\times10^2\times10^2\times10^2\times10^2$	
4^{th}			$=10^{2+2+2+2+2}$ $=10^{2+5}$	
5^{th}				

Table 5.6

Roll no.	Last digit of roll	day	Number of candy one gets	Multiplication form of number of candy one gets	Multiplication form of number of candy of all the members of a group	Product in exponential form of exponent
		1^{st}	1	1		
		2^{nd}				
		3^{rd}				
		4^{th}				
		5^{th}				

Now observe one thing, what we have expressed as exponential form of exponent may also be expressed as exponential form if we want. The table we get from the multiplication form of table 5.2 and 5.5 combined with last two columns is given below. The table has been filled partially.

Table 5.7

(The table has been filled partially with the given data in the table 5.2 and 5.5. Fill up rest of the table with the information you received.)

Multiplication form of number of candy of all the members of a group	Product in exponential form of exponent	Product in exponential form using the rule of product of exponent.

$1\times1\times1\times1\times1$	$(10^0)^5$	$10^0=1$
$10\times10\times10\times10\times10$	$(10^1)^5$	10^5
$10^2\times10^2\times10^2\times10^2\times10^2$ $\times10^2$	$(10^2)^5$	10^{10}

Similarly see, whether you can fill up the table below using data you have in the table 5.3 and 5.6 or not.

Table 5.8

(Fill up using data you have in the table 5.3 and 5.6.)

Multiplication form of number of candy of all the members of a group	Product in exponential form of exponent	Product in exponential form using the rule of product of exponent.

So tell, what have we found?

$10^2\times10^2\times10^2\times10^2\times10^2\times10^2$ can be written *as* $(10^2)^5$ and $(10^2)^5$ can be written as $(10^{2\times5}=10^{10}$

Task: 1)

Determine the following exponents.

Serial no.	Multiplication form of exponent.	Exponential form of exponent
1	$8^{14}\times8^{14}\times8^{14}\times8^{14}$	
2	$6^2\times6^2\times6^2\times6^2\times6^2\times6^2\times6^2\times6^2\times6^2\times6^2\times6^2$	
3	$14^3\times14^3$	
4	$18^9\times18^9\times18^9\times18^9$	
5	25^4	

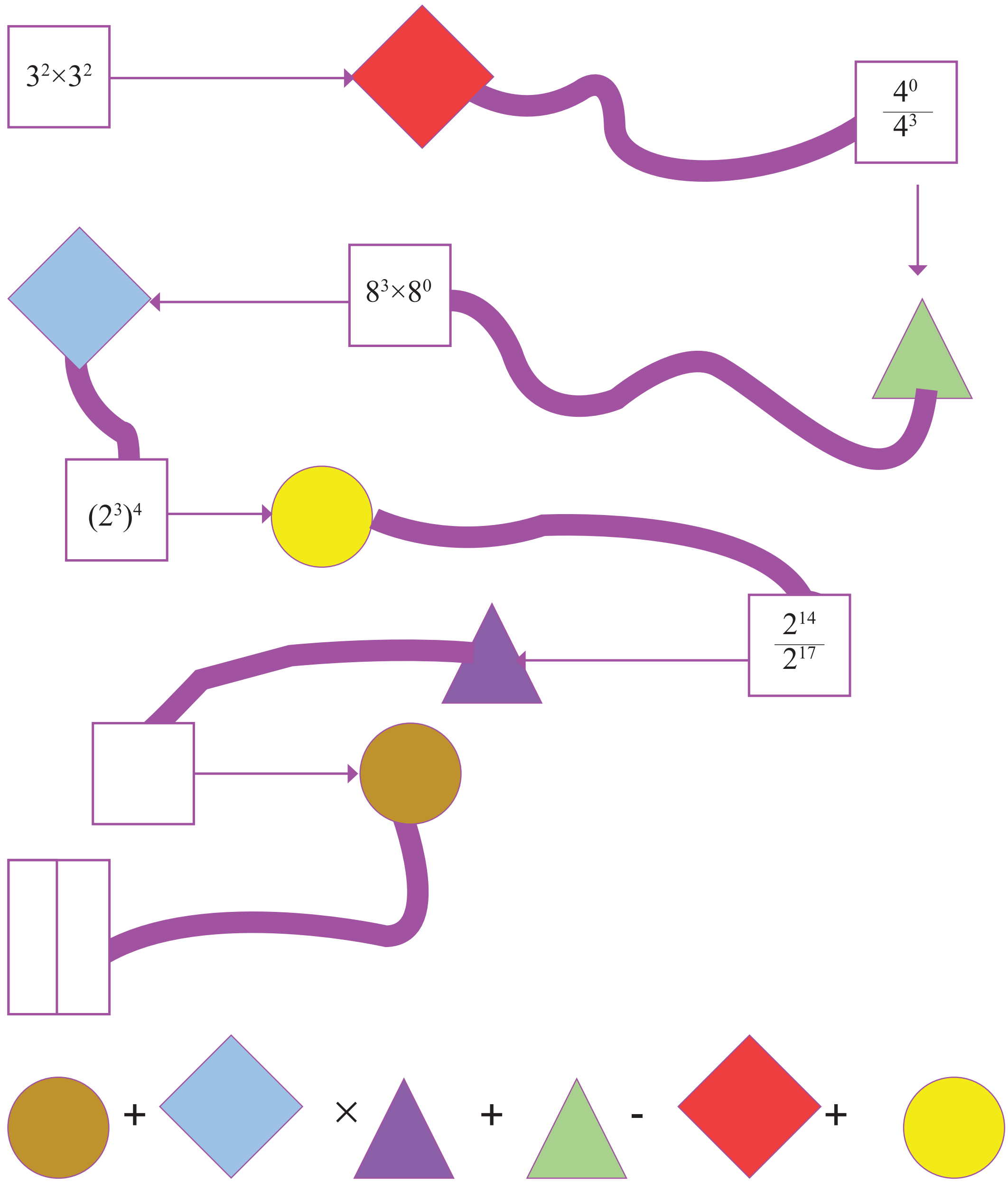
2) Determine the short forms of the following exponent.

Serial no.	Exponential form of exponent	Short form of exponent
1	$(43^{7})^{11}$	
2	$(99^{2})^{4}$	
3	$(34^{3})^{7}$	
4	$(2^{-2})^{3}$	
5	$(13^{3})^{1}$	

Individual work:

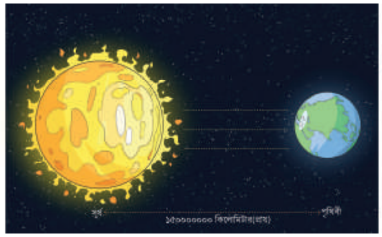
A credit card is like the picture. A credit card is usually used to purchase goods or pay bills. Credit card is a means of electronic money transaction like mobile banking. However anyone can't buy anything using this credit card. In that case there is a safety mechanism. That is the PIN. PIN is a combination of numbers only. It can't contain any letters or symbols except digits. If this PIN is not provided then no one will be able to avail the credit card benefits. That is, if the credit card owner forgets the PIN, then even he can't use it.

The father of Chobi has forgotten PIN of his credit card of bank. He can't remember it at all. Again it is very important to remember his PIN because he will buy essentials through the credit card. Then Chobi remembered that it is possible to find the PIN with the help of the diagram below.



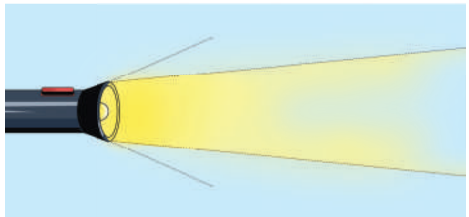
$3^2\times3^2$
$\frac{4^0}{4^3}$
$8^3\times8^0$
$(2^3)^4$
$\frac{2^{14}}{2^{17}}$
+
×
+
-
+

A little more on exponent



You know that it takes an average of 8 minutes and 18 seconds for light to reach the Earth from the Sun. But do you know how far the Earth is from the Sun? For convenience, it is assumed that the distance from the Sun to the Earth is 15,00,00,000 kilometres.

Task: Think and tell how about the velocity of light in words?



Again, do you know the velocity of light? For mathematical convenience, the velocity of light is assumed to be 30,00,00,000 metres per second.

Task: Think and express the distance from the Earth to the Sun in words.

We can express very large multiple form numbers easily and in small form with the help of exponents. Now think about it, whether we can take any help of exponents to express large numbers like distance of the Earth from the Sun or velocity of light in small form?

Let's see the matter. Let's work with the velocity of light. Observe the given table. Here are some cells filled up for you. You fill up the rest with the help of them and think about what exactly happens? But you must remember one thing while dividing and filling the table that there would never be any number less than 1 and without exponent in the second column.

Table7.1

Velocity of light: 30,00,00,000 metres per second.(Approximately)		
Number	Express it dividing by 10	Express in exponential form
300000000	30000000×10	30000000×10^1
	$3000000 \times 10 \times 10$	3000000×10^2
	$300000 \times 10 \times 10 \times 10$	300000×10^3

Thus it is not the case that only suffering can be reduced with the help of exponent. Rather much large numbers can be expressed in smaller forms.

So let's look at the table below to express the distance from the Sun to the Earth in small form. Here some cells are filled up for your convenience too.

Table 7.2

Distance from the Earth to the Sun is 15,0000000 kilometres (Approximate)		
Number	Express it dividing by 10	Express in exponential form
150000000	15000000×10	15000000×10^1
	1500000×10×10	1500000×10^2
	150000×10×10×10	150000×10^3
	15×	15×10

Here a point is that the last row of the table contains 15 together with 10 in exponential form. Now think about the previous table. We continue the dividing process until we get a number less than 10 but greater than 1. We can do that if we want. Complete it in the box below.

150000000		

So, what have you seen? What is obtained by expressing the distance from the Sun to the Earth in smaller form? So far we have seen the case of exponent of 10 in almost all cases. Now we will think about them. We try to see an example using number directly- 1 thousand. Its mathematical form is 1000.

1 thousand = 1000		
Number	Express it dividing by 10	Express in exponential form
1000	100×10	100×10^1
	10×10×10	10×10^2
	1×10×10×10	1×10^3

See, we get . We observed this very first. Think about it, is there any change if a number is in multiplication form with 1? No change. So we can write So, we may not write as implied number if it is 1 without exponent. So you have seen that various large real numbers can be expressed in small form in this way. Answer the questions below to narrate your understanding about the way of expressing from the above two examples.

* When will I finish the task of division?

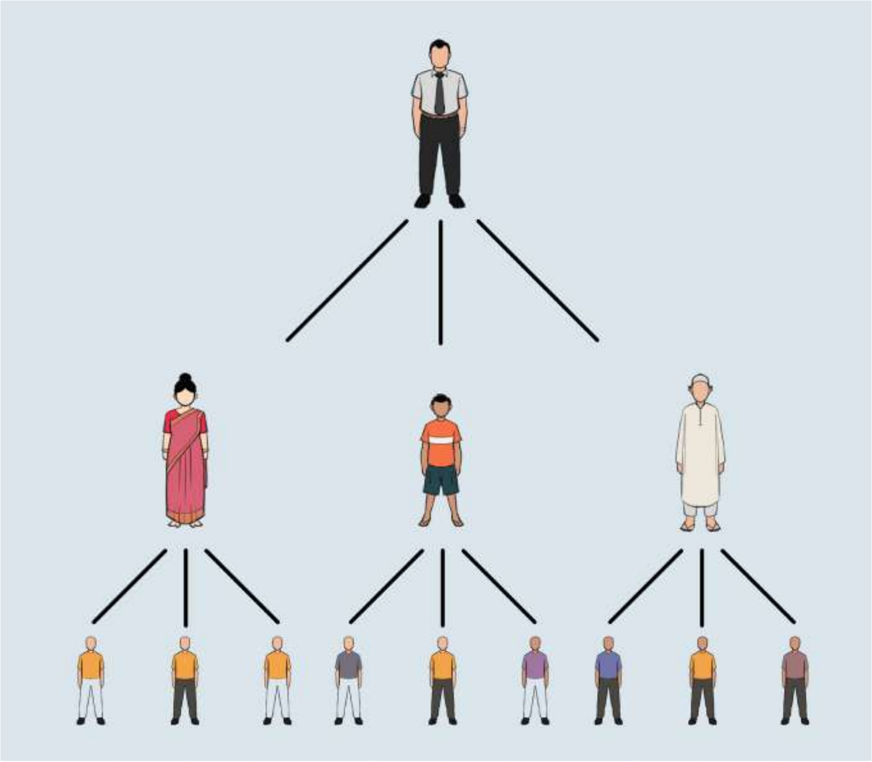
* Can the number without exponent in a division be less than 1? Or can it be equal to 1?

* Can the number without exponent in a division be equal to or greater than 10?

Task: The distance of the Moon from the Earth is about 3,84,000 kilometres. Express this distance in mathematically small size.

Individual Task:

1. You must be aware of the covid-19 pandemic. This deadly and contagious epidemic got the entire world to a standstill for a long time. We will try to enumerate one by one about that pandemic. Suppose there are three people in a house. All of them have been infected with covid. Now it is calculated that all three of them are able to infect at least three people separately in a day.



Apply the concept of exponent to find out the minimum number of covid-19 infected people in the next seven days if no hygiene measures are followed? Try to fill up according to the table. You can see the above tree-diagram for your help.

Day	Multiplication form of infected patients	Exponential form of infected patients
1st	3	3^1
2nd		
3rd		
4th		
5th		
6th		
7th		

In this way, at least how many infected patients will there be after 11th day or 14th day?

2) Fill in the blanks correctly.

Multiplication of exponent	Product (Exponential form)	Division of Exponent	Division	Eponential Form of exponent	Short form of exponent
$8^5 \times 8^{\square}$	8^{14}	$9^{58} \div 9^{\square}$	9^{21}	$(16^3)^{\square}$	16^{24}
$8^{\square} \times 8^{15}$	14^{22}	$11^{\square} \times 11^4$	11^8	$(26^{\square})^6$	26^{12}
$\square^{14} \times 5^{15}$	5^{29}	$35^{\square} \times 4^6$	4^{29}	$(\square^4)^{11}$	3^{44}
$\square^{10} \times \square^6$	17^{16}	$52^8 \div 52^{\square}$	52^0	$(5^4)^{-5}$	$5^{\square}$
$18^{21} \times \square^{67}$	18^{88}	$47^{21} \div 47^{\square}$	47^{-3}	$(15^{-7})^{-2}$	$15^{\square}$
		$19^{10} \div \square^{67}$	19^{-57}		

3) Express the numbers 10 thousand, 1 lac, 10 lacs, 1 crore and 10 crores in mathematically short form. Notice how many zeros are there after the 1. Now check the exponential form. Is there any relation with the exponent and the number of zeros after 1?

Exponent, Multiplication and Application of Formula of Algebraic Expressions

Exponent

Let's learn how to identify a Square

Let us take a square shaped paper. A square is a rectangle whose length and width are equal. Now fold the paper twice following the directions of the figure below (Once along the length and once along the width). The folds should be equal. Now when you straighten out the paper, you will find some small squares. Place a marble in each square. How many marbles do we need?

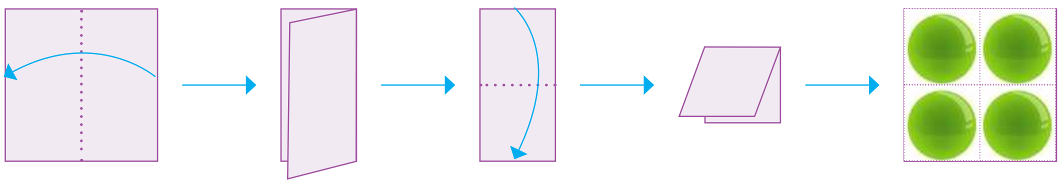


Figure-1

Similarly, let's fold another square shaped piece of paper equally into 3 parts along the length and the width. If you need, a scale can be used. Now place marbles in each of the squares. How many marbles do you need this time?

- Find out the number of marbles if the paper is folded four, five, six or even seven times along the length and width. Then fill-up the following table.

Table 1.1

Number of folds along the length and width	Number of Marbles	Number of folds along the length and width	Number of Marbles
2	4	5	
3		6	
4		7	

What did we learn from this experiment? The more we fold the paper the number of small squares is increasing. For instance, $2 \times 2 = 2^2 = 4$ Furthermore, think and answer this, if you had not folded the paper at all how many marbles would u need?

Individual Task: Take a square shaped piece and using a scale draw equally distanced eight line segments along the length as well as the width. How many small squares do you see?

Here, the number of small squares obtained through folding the square paper or the number of marbles can be called a square number or a perfect square. For example, when we fold a paper equally into 3 parts along the length and the width then we can place 3 marbles in each of the 3 rows. The total number of marbles are $3 \times 3 = 3^2 = 9$. Here, the number of rows and column are equal and therefore we say the square of 3 is 9, where 9 is a square number or a perfect square.

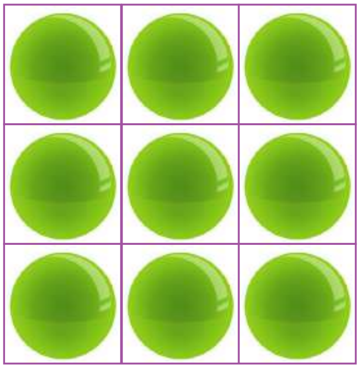


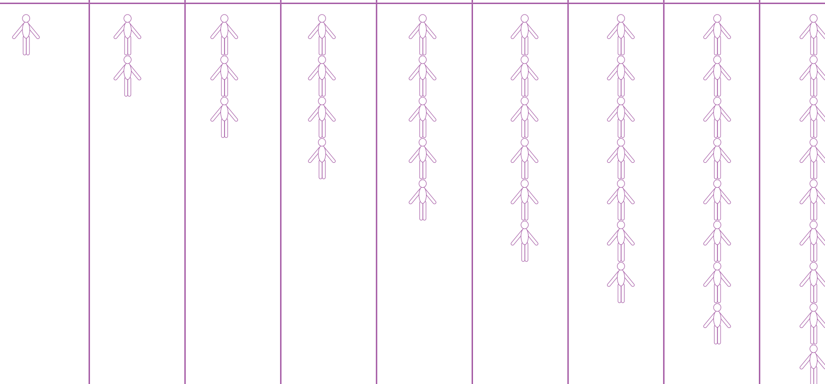
Figure-2

If we look at some numbers like 1, 4, 9, 25, 49 we can notice that they are also examples of perfect squares. On the other hand, 2, 5, 7, 12 are number which cannot be expressed as the square of an integer with itself. You justify this by using the folding and placing marble trick we did earlier.

Number	2	5	7	82	36	45	81	56	12
Is it a Perfect Square?									

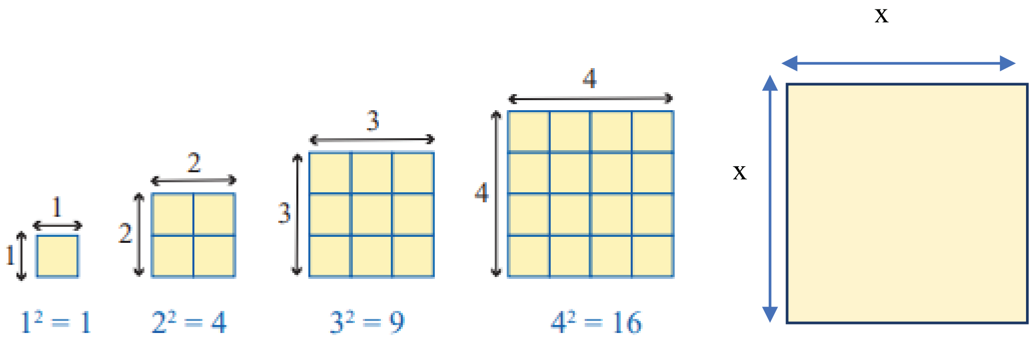
Team Work: Now that we have learnt about perfect squares, let us do a team task. Firstly, make 10 rows according to the unit digit of your class roll. Now, exchange places and try to make a square.

	0	1	2	3	4	5	6	7	8	9



What happens if you cannot figure out if the folds along the length and width are equal? We would not be able to play the marble game. Let us try to draw squares and figure out the area of it.

If we observe the following squares, can we tell the area of the last square whose length is x? Here x is an unknown expression.



We know, area of a rectangle = length × width.

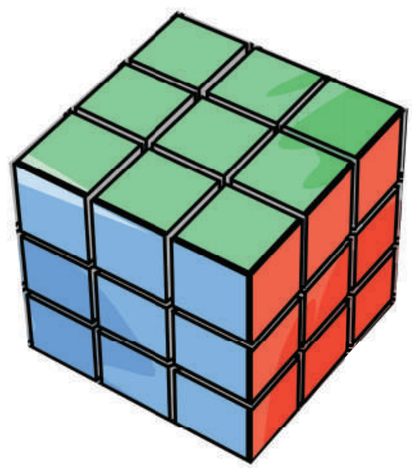
Now, a square is also a type of rectangle, whose length and width are equal. Surely, you can determine the area of a square,

Then, the area of the square is = length × width= $x.x = x^2$.

> We have seen that $3 \times 3 = 3^2$ is called the square of 3. Similarly $x \times x = x^2$ is called the square of x.

Cube:

We all are familiar with Rubik's cube. Look at the Rubik's cube with the dimensions 3 × 3 × 3, where there are 3 small cubes along the length, width and height. If you see the cube in depth, how many small cubes are there in total?



Let us try to construct a cube with dimensions 2 × 2 × 2 by using paper or wood. How many small cubes are needed to construct the following Rubik's cube?

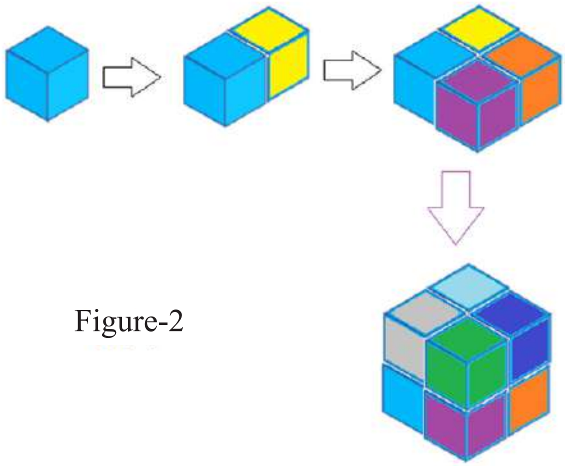


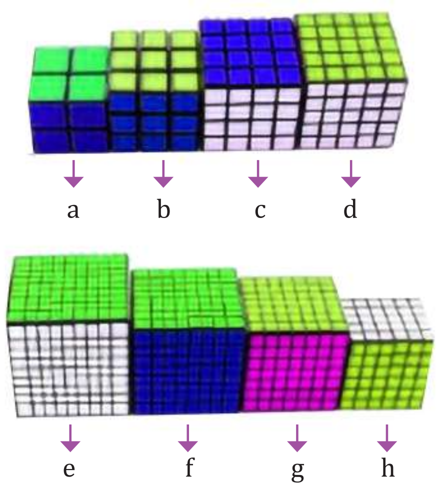
Figure-2

Individual work: Construct Rubik's cube with sides 3 and 4. How many small cubes are needed?

Observe that if we take 2 small cubes along the length and the width, then we need $2 \times 2 = 2^2 = 4$ small cubes for each sides. These 4 cubes have surfaces which are squares of length 2. Now if we take 2 more cubes along the height we can make another large cube, that needs a total of $4 \times 2 = 2 \times 2 \times 2 = 2^3 = 8$ cubes.

We have constructed a large cube with some number of small number of cubes. The number of small cubes needed is known as the perfect cube.

For example, when we are taking 2 cubes along the lenght, weight and height then to make the large cube we need $2 \times 2 \times 2 = 2^3 = 8$ cubes. In this case, we say 8 is the cube of 2, and 8 is a perfect cube.



Therefore, it can be said that to obtain the cube of a number we multiply the number by itself for three times.

In this manner, the cube of 3 is $3 \times 3 \times 3 = 3^3 = 27$.

Now fill up the following table: How many small cubes are needed to construct the following cubes?

Rubik's cube	Number of small cubes	Total number of small cubes are needed to construct the Rubik's cube	Rubik's cube	Number of small cubes	Total number of small cubes are needed to construct the Rubik's cube
a	2	$2 \times 2 \times 2 = 2^3 = 8$	e		
b	3	$\square \times \square \times \square = 3^{\square} = \square$	f		
c			g		
d			h		

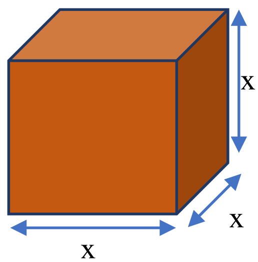
Remember that we compare with area to recognize the square of a variable. What can we do to compare with the volume to recognize the cube of a variable?

We all know that a cube with unit length has a volume 1 cubic unit.

We also know that the volume of a cuboid is= length × width × height.

However, a cube is a cuboid where all the sides are equal. That is, length = width = height.

Similarly, the volume of a cube from the figure = length × width × height $x.x.x = x^3$



We get from the above example that we have to multiply x three times to construct x^3

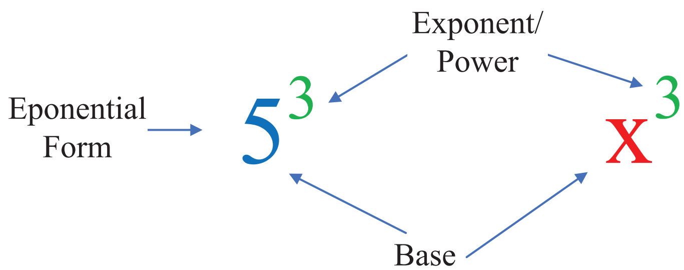
So, we can write in exponential form:

$$x^n = \underbrace{x \cdot x \cdot x \cdots x \cdot x \cdot x \cdot x}_{\text{Multiplying x n- times}}$$

Number of factors of x is n

The number of times of a factor in multiplication form of an expression is called exponent of that factor and the factor is called base.

Can you see any similarity between the exponent of a number and the exponent of a variable.



Individual task: Fill in the following table:

Repeated Multiplication	Base	Exponent	Power	Value
2.2.2.2.2	2	5	2^5	32
$x.x.x.x$				
4. 4. 4				
	5	3		
			6^2	

Have you noticed what happens when we multiply an exponential expression by another exponential expression? Let's see what happens when we multiply x^6 by x^3

$$\underbrace{(x.x.x.x.x.x)}_{x^6}.\underbrace{(x.x.x)}_{x^3} = \underbrace{(x.x.x.x.x.x.x.x.x)}_{x^9}$$

So, we can write, $x^6.x^3 = x^{6+3} = x^9$

Now fill in the following blanks:

$$\underbrace{(x.x.x.x.x.x)}_{x^{\square}}.\underbrace{(x.x.x)}_{x^{\square}} = \underbrace{(x.x.x.x.x.x.x.x.x)}_{x^{\square}} = x^{\square}$$

Therefore, if multiple exponential expressions of the same base are multiplied, base will remain unchanged and powers will be added. This **Multiplication Rule of**

Exponent is very important.

Multiplication Rule of Exponent

$$x^m . x^n = \underbrace{(x.x \ldots \ldots x)}_{\text{(m times x)}} . \underbrace{(x.x \ldots \ldots x)}_{\text{(n times x)}} = x^{m+n}$$

Let's see what happens when we divide x^7 by x^4. We express x^7 as 7 times multiplication in numerator. Similarly, we express x^3 as 3 times multiplication in denominator. Then we cancel out the appropriate number of terms from the denominator and numerator.

$$\frac{x^7}{x^3} = \frac{x.x.x.x.x.x.x}{x.x.x} = \frac{x.x.x.x}{1} = x^4$$

If bases are different, we have to cancel on the basis of different bases between denominator and numerator.

$$\frac{x^3y^5}{x^2y^3} = \frac{x.x.x}{x.x} . \frac{y.y.y.y.y}{y.y.y} = xy^2.$$

$$\frac{x^m}{x^n} = x^m \div x^n = \frac{\overbrace{(x.x\ldots\ldots x)}^{\text{(m times x)}}}{\underbrace{(x.x\ldots\ldots x)}_{\text{(n times x)}}} = x^{m-n}$$

Therefore, it can be seen that if more than one exponential expressions of the same base are divided then their base will be the same but power will be subtracted**. The Division Rule of exponent** is very important.

Individual task: Determine the values

1) $3^2 \times 9^2 =$, 2) $5^3 \times 25^{-2} =$, 3) $\frac{s^{13}}{s^5} =$, 4) $\frac{s^{13}t^{-4}}{s^5t^{14}} =$ 5) $\frac{2s^{13}t^{-4}}{4s^5t^{-14}} =$

Finding the Exponent of an Exponent:

Let, x^4 is an algebraic expression. You have to find the cube of it?

Now,

$(x^4)^3 = x^4 \cdot x^4 \cdot x^4$ 3 factors of x to the power 4.

$$\underbrace{(x\cdot x\cdot x\cdot x)}_{x^4}\cdot\underbrace{(x\cdot x\cdot x\cdot x)}_{x^4}\cdot\underbrace{(x\cdot x\cdot x\cdot x)}_{x^4}=\underbrace{(x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x)}_{x^{12}}$$

So, $(x^4)^3 = x^{12}$

If exponent is introduced upon an exponential expression, then the exponents are multiplied.

$(a^m)^n = a^{m\times n}$, where m, n are natural numbers and a is not zero. That is,

$$(x^m)^n = (x^m . x^m \dots x^m) = \underbrace{(\mathrm{x.x} \dots \mathrm{x})}_{(\mathrm{m}\ times\ \mathrm{x})} . \underbrace{(\mathrm{x.x} \dots \mathrm{x})}_{(\mathrm{m}\ times\ \mathrm{x})} \dots \underbrace{(\mathrm{x.x} \dots \mathrm{x})}_{(\mathrm{m}\ times\ \mathrm{x})} = x^{mn}$$

(Multiplying x $m.\mathrm{n}$ **times**)

Exponent rule of Product:

We can explain $(x^2y^2)^4 = (x^2)^4 \cdot (y^2)^4 = x^8y^8$ like the following.

$$(x^2y^2)^4 = \underbrace{(x.x.y.y)}_{x^2y^2}.\underbrace{(x.x.y.y)}_{x^2y^2}.\underbrace{(x.x.y.y)}_{x^2y^2}.\underbrace{(x.x.y.y)}_{x^2y^2}$$
$$= \underbrace{(x.x)}_{x^2}.\underbrace{(x.x)}_{x^2}.\underbrace{(x.x)}_{x^2}.\underbrace{(x.x)}_{x^2}.\underbrace{(y.y)}_{y^2}.\underbrace{(y.y)}_{y^2}.\underbrace{(y.y)}_{y^2}.\underbrace{(y.y)}_{y^2}$$
$$= (x^2)^4.(y^2)^4 = x^8.y^8 = x^8y^8$$

If same exponent is introduced upon the product of bases of exponential expression, the result will be product of distinct exponential expressions.

$$(xy)^n = (x^n . y^n) = \underbrace{(\mathrm{x.x} \dots \dots \mathrm{x})}_{(\mathrm{n}\ times\ \mathrm{x})} . \underbrace{(\mathrm{y.y} \dots \dots \mathrm{y})}_{(\mathrm{n}\ times\ \mathrm{y})} = x^n . y^n$$

Exponent rule of fraction:

Now we apply exponent upon a fraction. Let, $(\frac{x^3}{y^2})^4$, so we can write,

$$\left(\frac{x^3}{y^2}\right)^4 = \left(\frac{x^3}{y^2}\right)\cdot\left(\frac{x^3}{y^2}\right)\cdot\left(\frac{x^3}{y^2}\right)\cdot\left(\frac{x^3}{y^2}\right) = \frac{(x\cdot x\cdot x)\cdot(x\cdot x\cdot x)\cdot(x\cdot x\cdot x)\cdot(x\cdot x\cdot x)}{(y\cdot y)\cdot(y\cdot y)\cdot(y\cdot y)\cdot(y\cdot y)} = \frac{x^{12}}{y^8}$$

If there is same exponent of a quotient of bases, then the result will be the same exponent of both the denominator and numerator separately.

$$\boxed{\left(\frac{x}{y}\right)^n = \frac{\underbrace{(x.x\,\dots\dots x)}_{(\text{n } times\ \ x)}}{\underbrace{(y.y\,\dots\dots y)}_{(\text{n } times\ \ \text{y})}} = \frac{x^n}{y^n}}$$

Individual task: Determine the values

১. $(5^2)^3 =$ ২. $(a^{-4})^3 =$ ৩. $(3^3a^{-5}b^3)^3 =$ ৪. $\left(\frac{s^5}{3^4}\right)^3 =$ ৫. $\left(\frac{st^7}{rt^3}\right)^3 =$

Zero Exponent:

We know, $\frac{x^m}{x^n} = x^{m-n}$ from the exponent rule of fraction.

What will happen when $n = m$?

Let's see an example. Let, $\frac{x^4}{x^4}$. In that case,

$$\frac{x^4}{x^4} = x^{(4-4)} = x^0$$

But we know,

$$\frac{x^4}{x^4} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = 1$$

Rule of zero for exponent:

$$x^0 = 1,\ x \neq 0$$

If there is zero exponent upon any base, its value will be 1

$$x^0 = 1$$

(What is the value of $\frac{0}{0}$?)

Negative Exponent

We know, $\frac{x^m}{x^n} = x^{m-n}$ from the exponent rule of fraction. What will happen when n is greater than m? Let's see an example.

Let, $\frac{x^4}{x^6}$. In that case

That is, $\frac{x^4}{x^6}=x^{-2}$

$$\frac{x^4}{x^6} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x} = \frac{1}{x\cdot x} = \frac{1}{x^2}$$

We can briefly say, $\frac{x^6}{x^4} = x^2, \frac{x^4}{x^4} = x^0, \frac{x^4}{x^6} = x^{-2}$

Negative Rule of Exponent:

If a negative exponent is introduced upon a base, then exponent will be positive upon the reciprocal of base.

$$x^{-m} = (x^{-1})^m = \underbrace{(x^{-1}.\, x^{-1} \ldots \ldots x^{-1})}_{(\text{m } times\ x^{-1})} = \underbrace{(\frac{1}{x}.\frac{1}{x} \ldots \ldots \frac{1}{x})}_{(\text{m } times\ \frac{1}{x})} = \frac{1^m}{x^m} = \frac{1}{x^m}$$

Note that, $1^m = \underbrace{(1.1 \ldots \ldots 1)}_{(m\ times\ 1)} = 1$, Here m is a positive integer

Individual task: **Solve the following problem.**

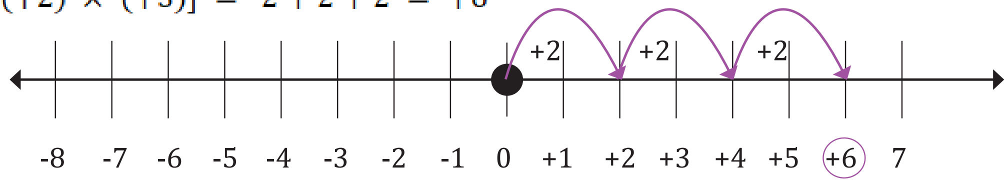
$(2a^{-2}b)^0$	$y^{-2}.y^{-4}$	$(a^{-5})^{-1}$	$s^{-2} \times 4s^{-7}$
$(3X^{-2}Y^{-3})^{-4}$	$(S^2T^{-4})^0$	$\left(\frac{2^{-2}}{x}\right)^{-1}$	$\left(\frac{3^9}{3^{-5}}\right)^{-2}$
$\left(\frac{s^2t^{-2}}{s^4t^4}\right)^{-2}$	$\frac{36a^{-5}}{4a^5b^5}$	$\frac{a^6b^7c^0}{a^5c^6}$	$\frac{a^{-6}b^7c^0}{a^5c^{-6}}$

Algebraic Multiplication

How the multiplication works in the number line is shown in the following examples. Here 4 problems are given. Each of them is expressed in the number line.

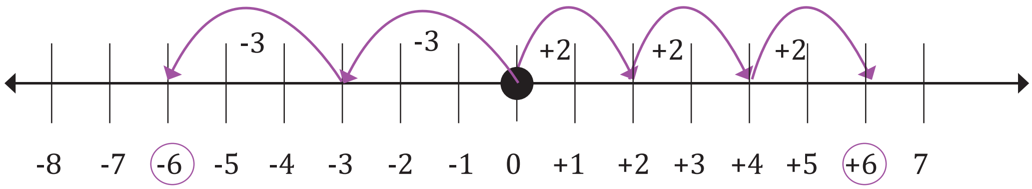
Now observe the following problems.

1.$[(+2) \times (+3)] = 2+2+2 = +6$



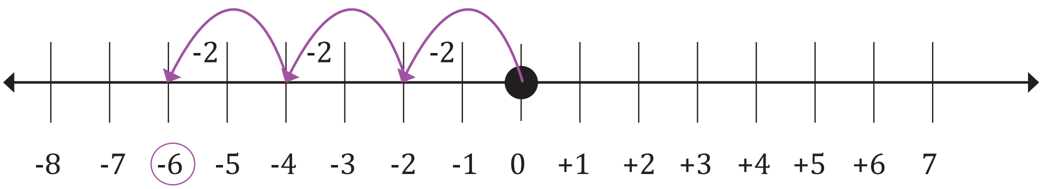
For No. 1 the position of the product is $(+6)$. In this case the position of product is shown on the right side of the number line.

2.$[(+2) \times (-3)] = (-3)+(-3) = -6$



We have seen in the case of no. 2 that the position of product in the number line is (-6). Here, the number has to move to the right on the number line for (+) sign of first number. Then the position of the product is left relative to (0) changing the direction for (-) sign before the product.

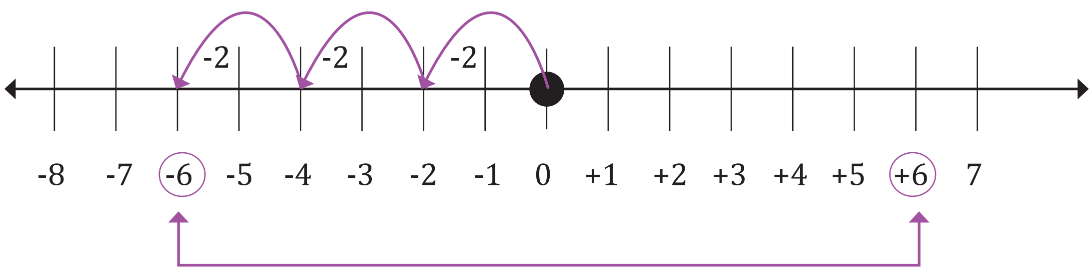
3.$[(-2) \times (+3)] = (-2) \times (+3) = (-2) + (-2) + (-2) = -6$



We have seen in the case of no.3 that the position of the product on the number line is (-6). Here the product is placed on the left side of the number line for (-) sign of first number.

4. $[(-2) \times (-3)] = -[(-2)+(-2)+(-2)] = -(-6) = +6$

6. -(-6)



We have seen in the case of no. 4 that the position of product in the number line is $(+6)$. Here, the first number moves to the left on the number line and then it has to move (+6) relative to (0) changing the direction for (-) sign before the product

1. $(+1) \times (+1) = +1$
2. $(+1) \times (-1) = -1$
3. $(-1) \times (+1) = -1$
3. $(-1) \times (-1) = +1$

Observe:

Product of two expressions with the same signs will be positive (+).

#Product of two expressions with opposite signs will be negative (-).

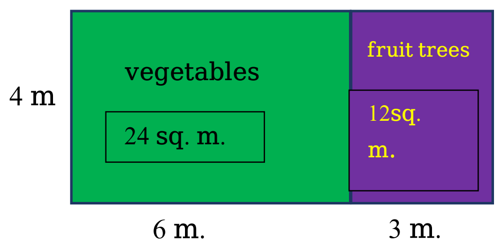
From above discussion, we can reach the following calculations

In the above discussion, you have learnt about the characteristics of product of numbers. Only positive signed numbers are used in arithmetic. However, both the positive and negative signed numbers and symbol of number are also used in algebra. In this chapter, we will learn about multiplication and division of algebraic expressions and exponent of multiplication and division of algebraic expressions.

Worksheet 1: School Garden Planning

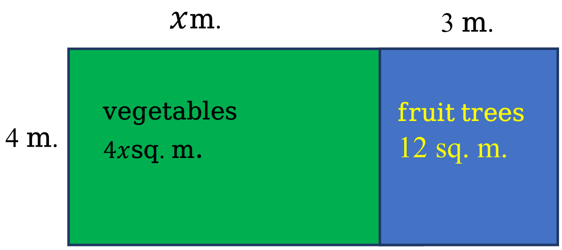
In order to beautify the environment of a school, the head of the institution decided to start a garden in the school yard. Some part of the garden was specified for vegetables cultivation and some parts for planting fruit trees. What will be the area of the garden? Let's make a possible plan.

In the beginning of planning, everyone takes a notebook and pen and makes the following paper/ drawn model of the garden and paint the vegetables and fruit garden parts with different colours. Determine the area of the garden.



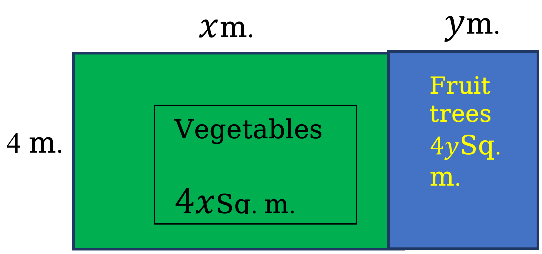
Here area of the garden $=4(6+3)$ sq. m. $= (4\times9)$ sq. m. $= 36$ sq. m.

The school authorty wants to change the length of the vegetables garden. So the length of the garden is changed by x . In this case, notice the change in the area of the garden



Here area of the garden = 4(x+3) sq. m. =(4x+12) sq.m.

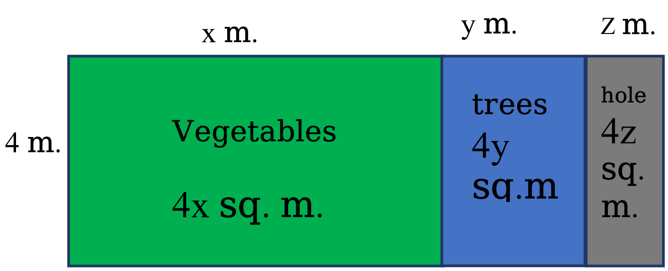
Now the length of the garden of fruit trees is changed by y, keeping the width fixed. As a result, what is the change in the area of the entire garden?



In this case, the area of the garden = $4(x + y)$sq. m. So here the area of the garden is expressed through algebraic expressions.

Individual Task: Make the model of this garden cutting paper.

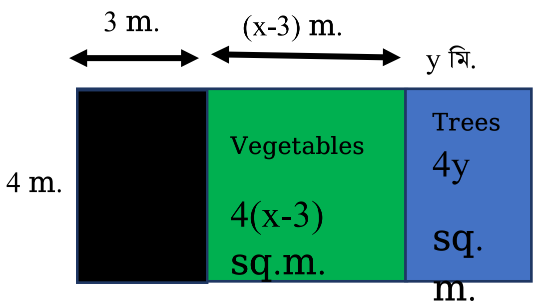
At this point let, the garden is divided into three parts to provide a hole to hold the water in the garden and the model is modified as follows.



In this case the area of the garden =4(x+y+z) sq.m. = (4x+ 4y+ 4z)sq.m.

So we can see that each case of the above $a(b+c)=ab+bc$, which is written as the product of algebraic expressions, indicates distributive law of multiplication.

Now the school planned to increase the corridor of the school building and so the length of the vegetables garden should be reduced by 3 metres. As a result, they changed the plan again as follows.

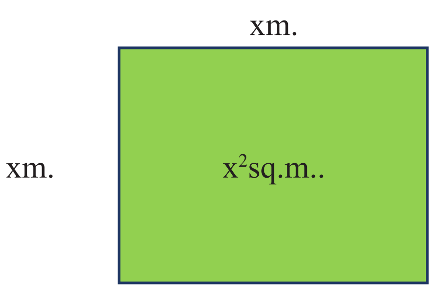


In this case, the area of the garden = $4(x-3) + 4y$ sq.m.

Individual work: In this case, you will make a new model by modifying the previous model.

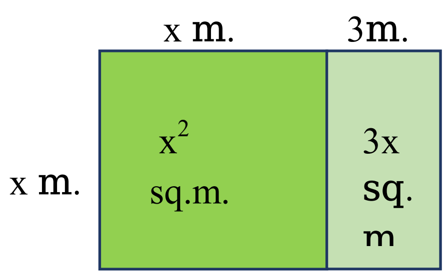
Worksheet 2- School Pond Excavation Plan

Now, to increase the income of the school, the head of the institution thought of digging a square pond next to the school ground for the purpose of fish farming. Students made a square model of paper with length x to determine the amount of land to dig a pond.



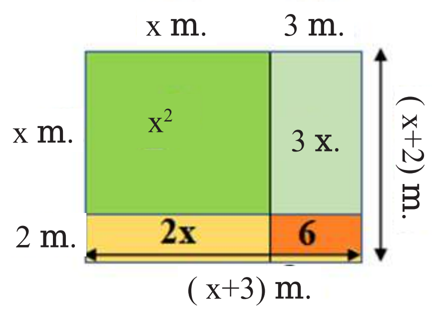
In this case, possible area of the pond=x^2sq.m.

Then students wanted to make the pond rectangular and made the model increasing the length of the pond by 3 metres as shown in the picture below.



In this case, area of the pond = $(x+3)x$ sq.m.= $x(x+3)$ sq.m.

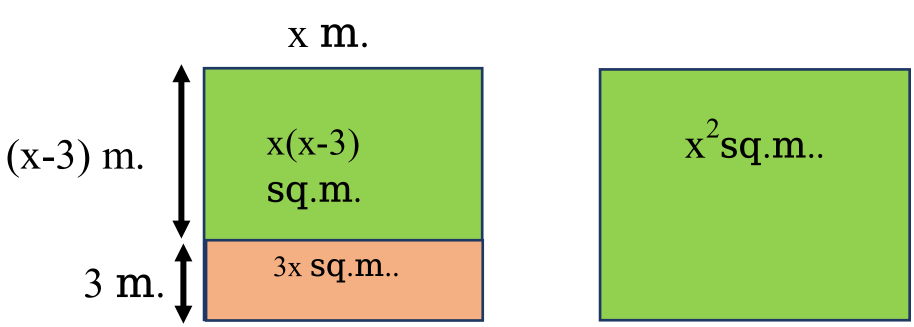
Then students increased the width by 2 m. and made a model like the following picture.



In this case, the area of the pond = $(x+3)(x+2)$ = x^2+5x+6 sq.m.

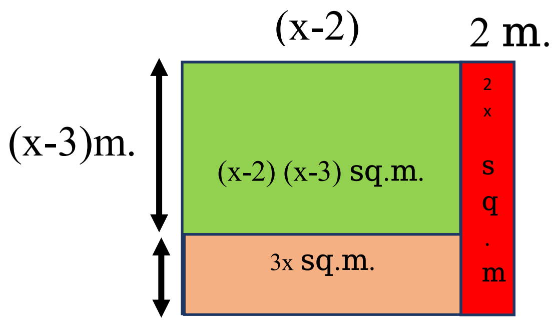
Individual task: Make the above model colouring white paper.

Then they made model decreasing the width by 3 m. like the following picture as an alternate of changing the area of pond.



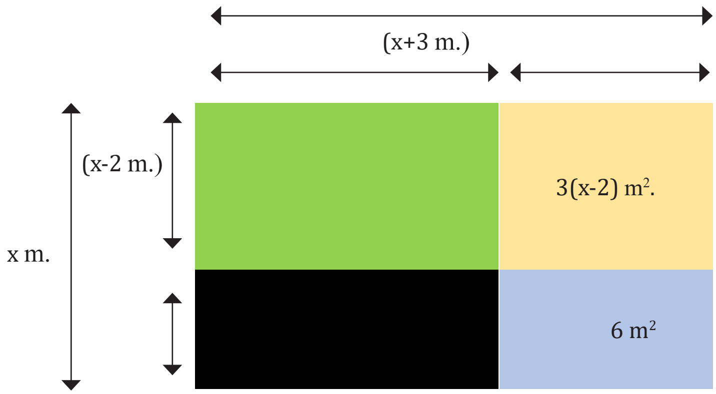
In this case, the area of the pond = $x\ (x-3)$sq.m. =x^2-3x sq.m.

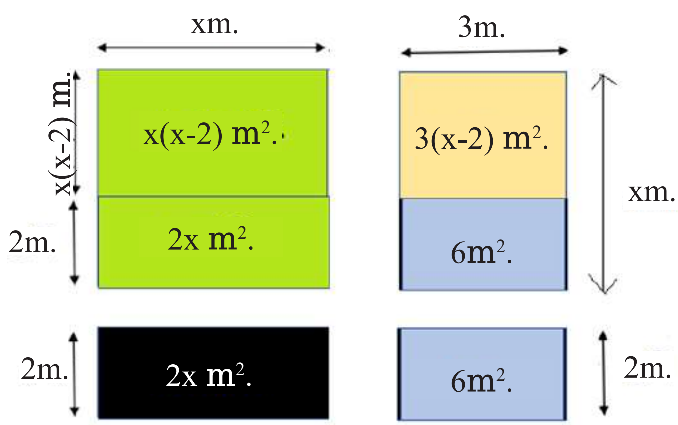
Again, model had been made decreasing the length by 2 m. of the pond like the picture below.



In this case, the area of the pond = $(x-2)\ (x-3)$sq.m. = $x^2 - 5x + 6$ sq.m.

Now, the students thought of a third alternative. They made a model, increasing length by 3 metres and decreasing width by 2 m. like the following.





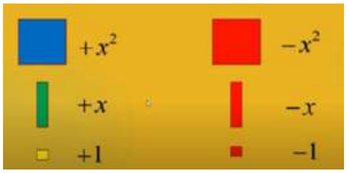
In this case, the area of the garden= $(x+3)\ (x-2)$sq.m.

Individual task: Make model of multiplication of algebraic expression cutting paper.

Observe the following example. Here method of multiplication of two algebraic expressions by paper cutting is shown.

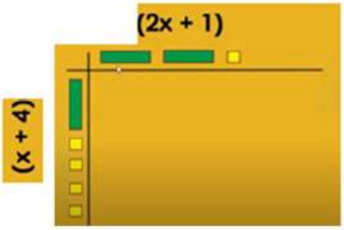
Example1: Determine the product of (x+4)(2x+1)

You make tiles cutting paper to determine the product as follows.

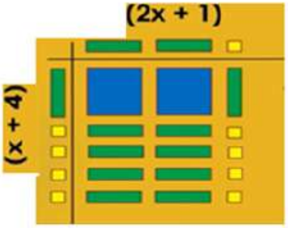


(Here adding a blue coloured part and a red coloured part)

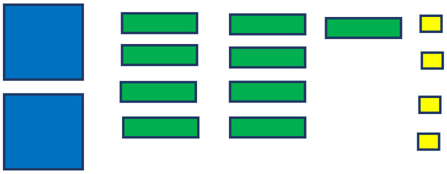
Place the factors as the size of tiles cutting paper like the following picture.



Now, multiply each paper tile of row part by each tile of column part and place that in the field of combination between row - column. As a result, you will get a rectangular area.



Add all the papers sequentially to get the area of rectangle. For example, =



$=2x^2+9x+4$

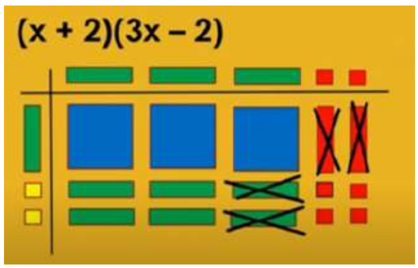
Individual task: Do the following example yourselves by cutting and multiplying the papers

Multiply with paper cutting: 2x+y-1, 3x

Multiplication using algebra tiles (2x+y-1)×3x)			
x	x	x	x
x	x^2	x^2	x^2
x	x^2	x^2	x^2
y	xy	xy	xy
-1	-x	-x	-x
	$6x^2+3xy-3x$		

Example: Arrange the two factors with paper to determine the area of rectangle whose sides are $(x + 2)$ and $(3x - 2)$like the following. In this case you can cancel positive and negative.

In this case,$(x + 2)(3x - 2) = 3x^2+6x - 2x - 4 = 3x^2+4x - 4$



Individual task: Multiply cutting paper: (x+3) (x+4)

Example: Determine the product:$(x + 3)(x - 7)$

	X	
x	x^2	3
-7	-7x	-21

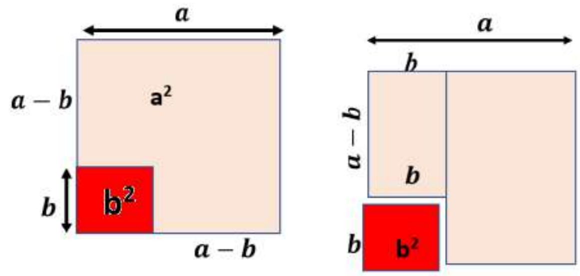
Arrange the term sequentially to get the product as follows

$(x+3)(x-7) = x^2 -7x+3x-21 = x^2-4x-21$

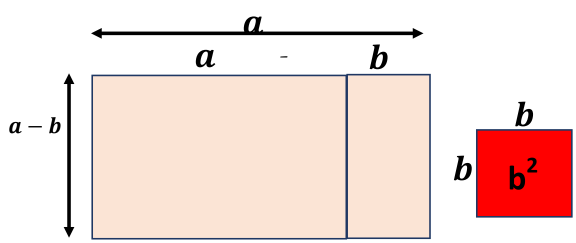
Individual task: Multiply with paper cutting: 2x+1) (x-2)

Worksheet 3: Determination of the area of (a+b) (a-b) (Paper model)

First take a white paper. Then draw a square whose sides are a. Colour it like the picture. Then draw another square with sides b on one corner of it and color that with red. Now cut out the small square (whose sides are b) from the big square (whose sides are a). So, the picture will be as follows.



The area of the resulting rectangle will be $= (\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b})$



Finally we get from the above picture

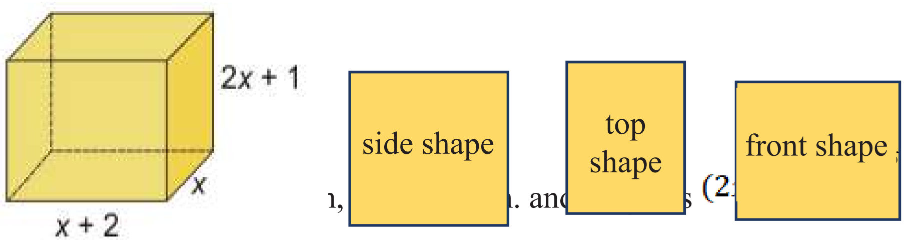
$$(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b}) = \boldsymbol{a}^2 - \boldsymbol{b}^2$$

Individual Work: Determine the product with paper cutting:: (a-b) (a-b)

This time the school authority arranges to install another water tank next to the garden as an alternative system to water the garden.

The students make a box cutting paper as in the picture below to determine the volume of a water tank whose length is $(x + 2)$ m. width is x m. and height is $(2x + 1)$ m. Then they determined the volume as follows:

Le



Volume of water tank =length × width × height cubic metre

$= (x+2).x.(2x+1)$ cubic metre

$= x\ (x+2)\ (2x+1)$ cubic metre

$= (x^2+2x)(2x+1)$ cubic metre

$= 2x^3 + x^2 + 2x^2 + 2x$ cubic metre

$= 2x^3 + 3x^2 + 2x$ cubic metre

Individual task:

<table>
<tr>
<td>

1.Determine the product cutting paper: $(x+2)(3x-2)$

</td>
<td>

3. Determine the product using formula::

I. $(x+y)(x-y)(x^2+y^2)$
II. $(a+1)(a-1)(a^2+1)$
III. $(x^2+xy+y^2)\times(x-y)$

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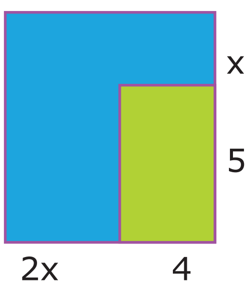
2. Determine the area of the picture below:



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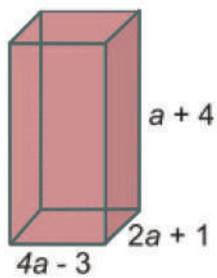
4. Determine the volume of the following picture:



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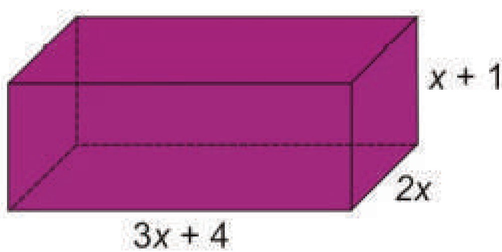
5. Determine the volume of the following picture:



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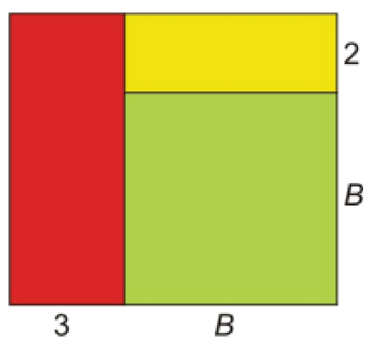
6. Determine the area of the following picture:



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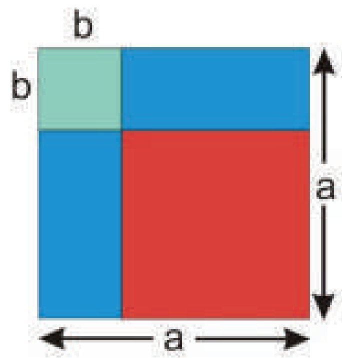
</table>

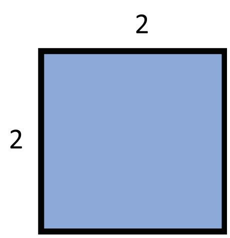
7. Determine the area of the orange coloured area of following picture:



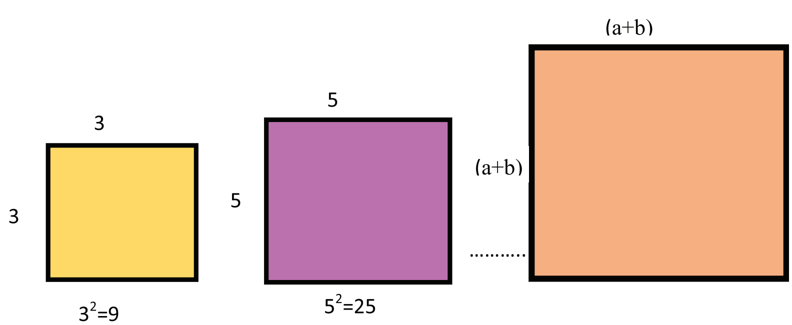
Algebraic formulae and applications (Square of binomial and trinomial expressions)

Square of binomial expression

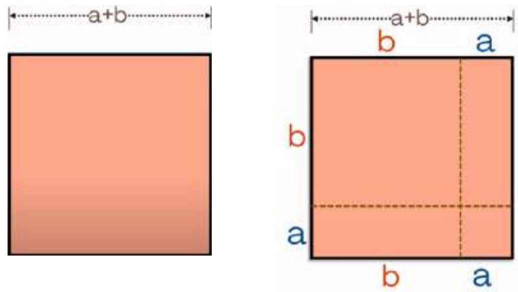
If you are asked what is the value of 2 multiplied by 2. You will surely tell 4. Now what is 3 multiplied by 3? You will surely tell 9. You have learnt it from the previous class. However, if you are asked, what is the area of a rectangle of length 2 cm. and width 2 cm. Now you will surely draw a rectangle of length 2 cm. and width 2 cm. Your drawing will be as follows.



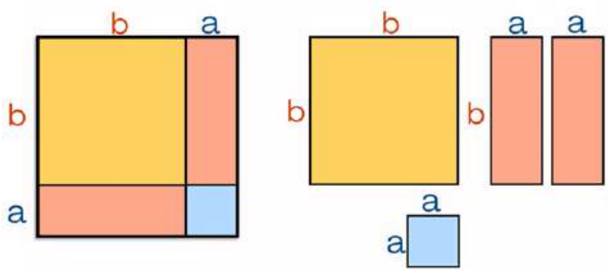
Again if length is 3 cm. and width is 3 cm, then what will be the area? If length is 5 cm. and width is 5 cm. then what will be the area? If length is $(a+b)$ cm. and width is $(a+b)$ cm. then what will be the area? Let's observe the following pictures.



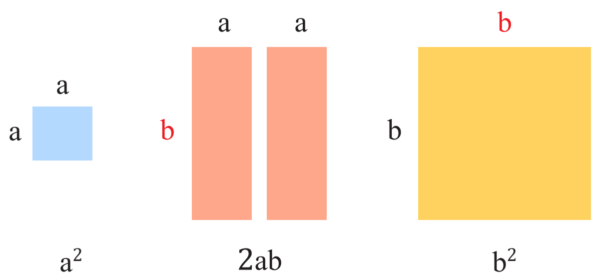
Let's try to find the value of $(a+b)^2$. First take a square paper. Identify the sides a and b from that like the following picture. As a result, four fields will be identified.



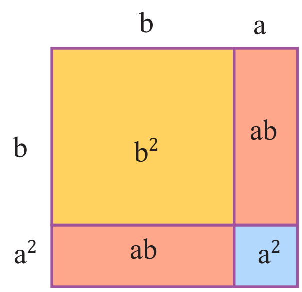
Cut the fields and separate them. Four areas of the fields will be available like the picture below.



Sorting the separated fields, we will get something like the following picture



Finally we will get from the following picture.



$$(a+b)^2 = a^2+2ab+b^2$$

Let's now verify the validity of this formula. In this case, we will utilize a geometric method. Let, a=3 and b=2, then,

$(a+b)^2=a^2+2ab+b^2$

Now draw a square with side length of (a+b), that is, (3+2)s

The whole area of the square will be $(3+2)^2=5^2=25$

The area of a square with side length 3 unit = 9 sq. unit.

The area of a square with side length 2 unit = 4 sq. unit.

The area of a rectangle whose two sides are 2 unit and 3 unit = 6 sq. unit.

The area of a rectangle whose two sides are 3 unit and 2 unit = 6 sq. unit

In this case also, whole area of the square will be =(9+4+6+6) sq. unit = 25 sq. unit.

Since, in both the cases the value of the area is equal, so we can tell, $(a+b)^2= a^2+2ab+b^2$

Individual task: Construct squares with the help of picture like the above.

1. (m+n)	4. 105
2. (4x+3)	5. 99
3. (3x+4y)	

Individual task: Prove it with paper cutting: $a^2+b^2=(a+b)^2-2ab$

Constructing Perfect Squares easily with the Algebraic Formula:

By now, we all are acquainted with the algebraic formula of determining squares, $(a+b)^2=a^2+2ab+b^2$. Then, lets calculate the square of 42.

$$42^2=(40+2)^2=40^2+2\times40\times2+2^2=1600+160+4=1764$$

Task: Calculate the squares of 52, 71, 21, 26, 103 using the above method.

Now, the you will find the values of the squares of the numbers 1-20 in the following table. Using the algebraic formula fill in the gaps in the table.

Table 1.2

Number	Square	Number	Square	Number	Square	Number	Square
1		6		11	121	16	
2	4	7		12		17	
3	9	8		13		18	
4		9	81	14		19	361
5		10		15		20	

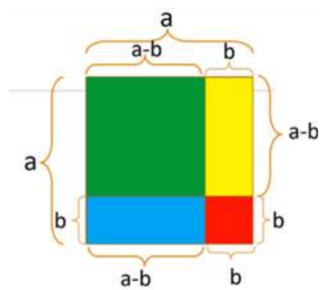
Look at the unit digits of the entries of the table , do you see any patterns or similarities?

Task:

A number can be a perfect square if the unit digits are what numbers?
Write five numbers who are not squares, determine this by looking at the unit digit.

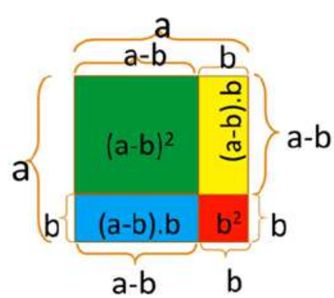
Geometrical proof of formula $(a-b)^2$

First, we cut a square size paper to determine the area of the square. Then identify the sides with 'a' and 'b' like the following picture. And find the answer to the following questions.



- What is the side length of the green square?
- What is the area of the green square?
- What is the area of the yellow rectangle?
- What is the area of the red square?
- What is the area of the blue rectangle?

Have you found the answer of the above questions? Then match the picture.



- Side length of the green square = $(a-b)$
- Area of the green square = $(a-b)^2$
- Area of the yellow rectangle = $(a-b)b$
- Area of the red square = b^2
- Area of the blue rectangle = $(a-b)b$

Now, area of the green square = Area of whole square [Area of the yellow rectangle + Area of the red square + Area of the blue rectangle.] That is,

$(a-b)^2 = a^2-\{(a-b)b+b^2+(a-b)b\}$

$= a^2-\{ab- b^2+b^2+ab-b^2\}$

$= a^2-\{2ab-b^2\}$

$= \ (a-b)^2$

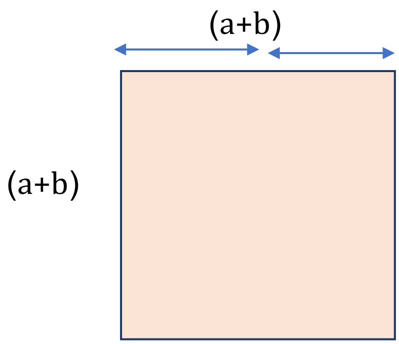
$=a^2-2ab+b^2$

Individual task: Determine the square with the help of pictures like the above.

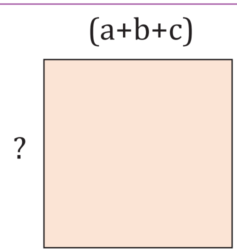
1. (m+n)	4. 95
2. (4x+3)	5. 99
3. (3x+4y)	

Individual Task: Find the Square of Trinomial Expressions.

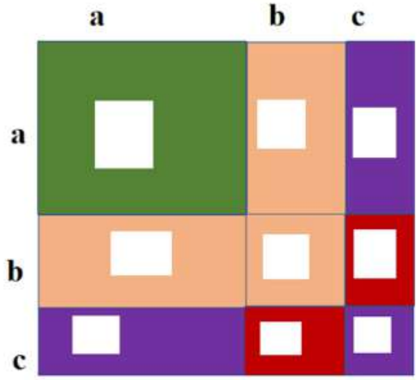
In the meantime we have learnt the proof of $(a+b)^2$. In that case the square was as follows



But if one side of the square is (a+b+c) unit, then the picture will be as follows,

	$(a+b+c)^2=?$

Let's try to find the value of $(a+b+c)^2$, which is the area of a square whose side length is (a+b+c). First take a square size paper. Identify the sides a, b and c like the picture below. Then the square will be divided into ☐ parts by the sides.



Now, write the area values from the above figure in the gap boxes.

Here, length of the one side of the square = ?

Now, from the figure, the sum of the area of all of the fields =?

We get from the above picture, $(a+b+c)^2 = ?$

Individual work: Solve the following problem using paper cutting of picture drawing.

Determine the square of $(2x+3y+4z)$.

Individual task: Determine the square of the following expressions by cutting paper and submit to teacher.

1. $a+3$ 2. $3x-5$ 3. 999 4. $2x+y+3z$

Prove it by cutting paper

1. $a^2+b^2=(a-b)^2+2ab$
2. $(a-b)^2=(a+b)^2-4ab$
3. $(a+b)^2=(a-b)^2+4ab$
4. $(a+b)^2+(a-b)^2=2(a^2+b^2)$
5. $(a+b)^2-(a-b)^2=4ab$

GCD and LCM of Ordinary Fractions and Decimal Fractions

You all are familiar with GCD. GCD means Greatest Common Divisor. Here, try to remember two things. You need to compare between at least two numbers to determine the GCD. Compare what? Their multipliers or factors. Now imagine, what the multipliers or factors are? Or, what are the common multipliers? You have seen that for positive whole numbers, you may find one or more multipliers or factors or divisors of two separate numbers, which are multipliers or divisors of both the numbers. Those multipliers are called **common divisors/multipliers**. Later, the largest common multiplier among them will be called the **Greatest Common Divisor / Multiplier.** What if there is only one common divisor/multiplier? Now it is possible to find GCD of fractions, along with positive whole numbers.

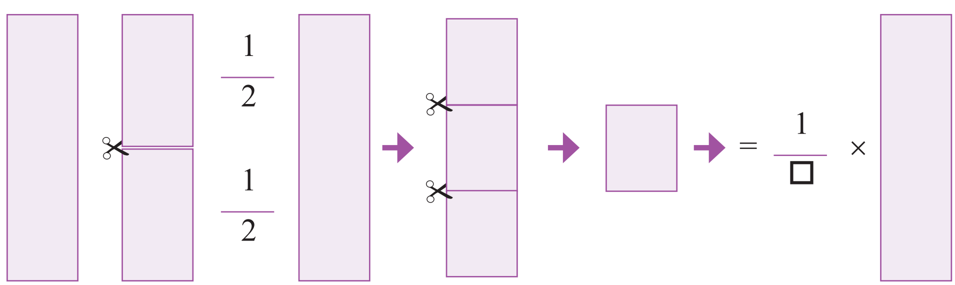
Divisors/Multipliers of Ordinary Fractions

First think about the divisors/multipliers of fractions. First let us compare the divisors of whole numbers. Here we know that, divisors of a whole number are those whole numbers which divide the whole numbers completely, without any remainder. For example, consider the number 12, 12 is completely divisible by the numbers 2, 3, 4, 6 and 12. Do we get a whole number if 12 is divided by 5? The answer is, no.

Task: What are the divisors of 18?

Now what will happen in case of an ordinary fraction? Let us play a game first.

Look for divisors:



First take a piece of paper.

Cut the paper into two equal parts. So what part is one of the cut piece of that paper? The cut piece will be $\frac{1}{2}$ of the original paper. Now take 3 more papers, and divide them into 3, 4 and 5 pieces respectively and fill up the table below.

Table 1.1

Number of equal parts	What part 1 piece of whole part
2	$\frac{1}{2}$
3	
4	
5	

Observe, we obtained few fractions from the table. Now take one piece from the equally divided 2 pieces. We can call the piece $\frac{1}{2}$. Now think, can you fold this piece into two equal parts? See, of course you can fold it. Now think, what part is each of two portions obtained, of the piece? Easily can be said that this is also $\frac{1}{2}$ of the piece. But the piece itself is a $\frac{1}{2}$. Then what part each of these pieces will be of the original paper? Easily seen it is $\left(\frac{1}{2} \div 2\right) = \frac{1}{4}$. Now think, instead of 2 folds, if 3 folds are done, what each piece will be of the original paper? Or, when equal 4, 5 and 6 folds are given, what each part will be of the original paper? Fill that up in the table below.

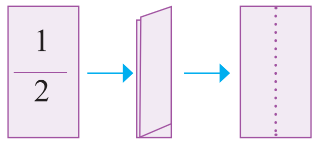


Table 1.2

(partially filled. Fill it up through your task)

Fraction (piece is part of original paper)	no. of equal fold	Division process	After fold, quotient obtained, part of original paper
$\frac{1}{2}$	2	$(\frac{1}{2} \div 2)$	$\frac{1}{4}$
	3	$(\frac{1}{2} \div 3)$	
	4		
	5		
	6		

Now what do you see? Each time you folded the paper into whole number of times, but with respect to that, you are getting a fraction.

Task: Take one piece of the paper from the 3, 4 and 5 equal pieces of the paper, as in Table 1.1 and for each one, make a table like Table 1.2 and fill it up.

Now, think about, what we are getting through folding like this? Let's start thinking about that $\frac{1}{2}$ piece from the example above. Folding that $\frac{1}{2}$ piece into 3 equal pieces, means dividing that by 3. In this game of folding the paper we are actually dividing a fraction by a whole number. That means, we are dividing the fraction with the whole number of folds. What are we actually getting in this manner? The multipliers/factors of are being determined in this way. Then what are the multipliers/factors of a fraction? The fraction or a whole number we get by dividing a fraction with a whole number is the multiplier/factor of that fraction.

Now think about a matter! What can be the meaning of folding that $\frac{1}{2}$ piece into 1 part? That means there is not actually any folds! That piece of paper remains unfolded. What does that mean? That fraction itself is a multiplier/factor of that fraction. Because 1 is also a whole number. So, dividing by 1 also gives a whole number or a fraction.

Now let us do a task. Let us prepare a table of multipliers/factors by filling up the following table. Find the first 10 multipliers/factors of each fraction. The table is partially filled up.

Table 1.3

Fraction	Multipliers/Factors	Fraction	Multipliers/Factors
$\frac{1}{2}$	$\frac{1}{2}$, $\frac{1}{4}$,	$\frac{1}{4}$	
$\frac{2}{3}$		$\frac{4}{5}$	
$\frac{1}{3}$		$\frac{1}{5}$	
$\frac{3}{4}$		$\frac{3}{5}$	

Note that, following this system, you can determine the multipliers/factors of any fractions you want.

Task: Take 5 common fractions according to your choice and find 10 multipliers/factors of each.

Now think about the fact that, each time we determined 10 multipliers/factors. Now we want to find all the multipliers/factors. Now try to find all the multipliers/factors of the fraction $\frac{1}{2}$ and write them down in your exercise book.

Could you determine all the multipliers/factors? If you count, you will notice that you can never find all the multipliers/factors. Hope you understand the reason. Very simply said, there are infinitely many whole numbers. So, you can divide a common fraction by infinitely many whole numbers. And what do we know about rational numbers? If a common fraction is divided by a whole number, then that must be expressible as a ratio or fractional form. That means, the number of multipliers/factors are not of fixed numbers, like the multipliers/factors of whole numbers. The multipliers/factors of fractions are infinitely many.

Common Multipliers / Factors of common Fractions:

So far, we have learnt about the multipliers / factors of common fractions. Now we shall try to learn about common multipliers/factors of several common fractions.

We try to learn that with an example. Consider two fractions $\frac{1}{6}$ and $\frac{1}{8}$.

Now we shall find 10 multipliers of these two fractions; these are written in the table below:

Table 2.1

fraction	Multipliers/factors
$\frac{1}{6}$	$\frac{1}{6}, \frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{30}, \frac{1}{36}, \frac{1}{42}, \frac{1}{48}, \frac{1}{54}, \frac{1}{60}$
$\frac{1}{8}$	$\frac{1}{8}, \frac{1}{16}, \frac{1}{24}, \frac{1}{32}, \frac{1}{40}, \frac{1}{48}, \frac{1}{56}, \frac{1}{64}, \frac{1}{72}, \frac{1}{80}$

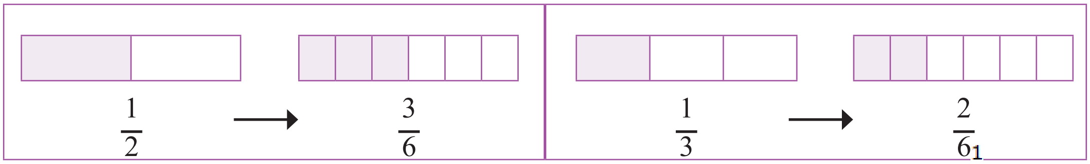
Identify the fractions which occur on both the rows in the above table. Easily, you can see that $\frac{1}{24}$ *and* $\frac{1}{48}$ occur in both the rows. So, from idea of multipliers / factors of whole numbers, it is possible to say in this case that, $\frac{1}{24}$ *and* $\frac{1}{48}$ are the common multipliers/factors of $\frac{1}{6}$ *and* $\frac{1}{8}$.

Task: Determine the common multipliers/factors of the following fractions, by finding 10 multipliers of each.

1) $\frac{1}{2}$ and $\frac{1}{3}$ 2) $\frac{1}{3}$ and $\frac{1}{4}$ 3) $\frac{1}{3}$ and $\frac{1}{10}$

Now let us get back to our previous example. There we obtained the multipliers/ factors were $\frac{1}{24}$ *and* $\frac{1}{48}$. Now can you tell which one is bigger?

You have learnt to make fractions with the same denominator. For example, we can do that, if we want, with the two fractions $\frac{1}{2}$ *and* $\frac{1}{3}$. What do we have to do if we want to make them fractions with the same denominator? Since we are talking about same denominator, we have to make the denominator of the two fractions same. What will it be in this case? Both the fractions will have denominator 6 since the LCM of 2 and 3 is 6. Now you have to think about the numerators of both the fractions. So what will be the new numerators of the fractions having same denominators? In this case, we can think about taking help of grids.



What can you see from the grid? If the denominator is 6, the numerator of $\frac{1}{2}$ will be 3. The reason is very easy. It is seen from the grid that if the denominator is 6, the basic grid is divided into 6 parts. Here the number of yellow-coloured parts is 3. That means if the number of parts is 6, number of yellow-coloured parts is 3. That means, the numerator increases as with same multiples as the denominator. Hence, numerator will be $(1 \times 3) = 3$. So, the new fraction is $\frac{3}{6}$.

Similarly, the denominator of $\frac{1}{3}$ is now 6. Hence the fraction is $\frac{2}{6}$.

Now it is easy to understand that $\frac{3}{6} > \frac{2}{6}$; that means, $\frac{1}{2} > \frac{1}{3}$.

Task: 1) Use grids to determine which is the bigger fraction among $\frac{2}{5}$ and $\frac{4}{7}$

2)Use grids and determine which is the bigger fraction among $\frac{1}{24}$ and $\frac{1}{48}$

So, think which is the bigger fraction among $\frac{1}{24}$ and $\frac{1}{48}$?

Here, the LCM of the two denominators is 48. So, from the idea of the equal denominator, it is easy to determine that $\frac{1}{24} = \frac{2}{48}$. That means $\frac{2}{48} > \frac{1}{48}$. That means $\frac{1}{24}$ is bigger. So, among the two fractions, $\frac{1}{24}$ is the largest common multiplier or the greatest common divisor or GCD.

Task: You determined the common multiplier of the following fractions. Now find the GCD:

1) $\frac{1}{2}$ and $\frac{1}{3}$ 2) $\frac{1}{3}$ and $\frac{1}{4}$ 3) $\frac{1}{3}$ and $\frac{1}{10}$

Now think about the two fractions we worked with so far, the numerators of which were just 1. Now think a little bit of something different. Consider the fractions $\frac{1}{4}$ and $\frac{2}{5}$. Determine the GCD of them.

Let us first try to find the multipliers/factors of them according to the rules of finding GCD. What are the multipliers/factors of $\frac{1}{4}$? We also know that it is not possible to find all the multipliers. So, try to find 10 multipliers/factors.

Table 2.2

Fraction	Multipliers/Factors
$\frac{1}{4}$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{12}, \frac{1}{16}, \frac{1}{20}, \frac{1}{24}, \frac{1}{28}, \frac{1}{32}, \frac{1}{36}, \frac{1}{40}$

Now think, what will the multipliers/ factors be of $\frac{2}{5}$?

Let us try to find them.

Table 2.3

Fraction	Whole no.	Division process to find multipliers	Simplest form of multipliers	Whole numbers	Division process to find multipliers	Simplest form of multipliers
$\frac{2}{5}$	1	$\left(\frac{2}{5} \div 1\right) = \frac{2}{5}$	$\frac{2}{5}$	6	$\left(\frac{2}{5} \div 6\right) = \frac{2}{30}$	$\frac{1}{15}$
	2	$\left(\frac{2}{5} \div 2\right) = \frac{2}{10}$	$\frac{1}{5}$	7	$\left(\frac{2}{5} \div 7\right) = \frac{2}{35}$	$\frac{2}{35}$
	3	$\left(\frac{2}{5} \div 3\right) = \frac{2}{15}$	$\frac{2}{15}$	8	$\left(\frac{2}{5} \div 8\right) = \frac{2}{40}$	$\frac{1}{20}$
	4	$\left(\frac{2}{5} \div 4\right) = \frac{2}{20}$	$\frac{1}{10}$	9	$\left(\frac{2}{5} \div 9\right) = \frac{2}{45}$	$\frac{2}{45}$
	5	$\left(\frac{2}{5} \div 5\right) = \frac{2}{25}$	$\frac{2}{25}$	10	$\left(\frac{2}{5} \div 10\right) = \frac{2}{50}$	$\frac{1}{25}$

We can easily see that after finding 10 multipliers for each, the two fractions have only one common multiplier, which is $\frac{1}{20}$. The question is, can we call this the GCD? Common fractions do not have fixed number of multipliers like whole numbers have. Hence, the number of common multipliers is also not fixed. This means that the common multipliers among several fractions are also infinite.

Can you understand something through your work so far? Is there any relationship between the common multipliers/factors ? If we consider the two examples shown earlier, $\frac{1}{6}$ and $\frac{1}{8}$, we shall see that the common multipliers of the two fractions were $\frac{1}{24}$ and $\frac{1}{48}$. Let us now do a bit more work on it. We shall now find 12 multipliers of the two fractions.

Table 2.4

Fraction	Multipliers
$\frac{1}{6}$	$\frac{1}{6}, \frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{30}, \frac{1}{36}, \frac{1}{42}, \frac{1}{48}, \frac{1}{54}, \frac{1}{60}, \frac{1}{66}, \frac{1}{72}$
$\frac{1}{8}$	$\frac{1}{8}, \frac{1}{16}, \frac{1}{24}, \frac{1}{38}, \frac{1}{40}, \frac{1}{48}, \frac{1}{56}, \frac{1}{64}, \frac{1}{72}, \frac{1}{80}, \frac{1}{88}, \frac{1}{96}$

What do you notice here? Did we get any new multipliers? From the list, you can see that $\frac{1}{72}$ is also a multiplier of both the fractions .

Now think about something. We can also find a relation between the common multipliers. Observe below,

$\frac{1}{24} \div 1 = \frac{1}{24}$	$\frac{1}{24} \div 2 = \frac{1}{48}$	$\frac{1}{24} \div 3 = \frac{1}{72}$

That is, determining the multipliers sequentially, using the first common multiplier obtained, it is possible to obtain the other common multipliers. The way we determined common multipliers of fractions by division, similarly, sequentially dividing the first common multiplier obtained, by whole numbers, the other common multipliers can be determined.

Now think, when you folded the paper, which among the two was bigger the paper or the parts obtained by folding? Of course, the paper was bigger as the same paper was being folded again and again. It is clear from here that, when a fraction is divided by another whole number, the new quotient or fraction obtained is definitely smaller compared to the original fraction.

So, what do you understand from here? If the common multipliers are determined sequentially, then the first common multiplier will be the Greatest Common Multiplier/ Factor/Divisor or GCD. Now what can you say? The only common multiplier/factor/ divisor obtained by finding the 10 multipliers of $\frac{1}{4}$ and $\frac{2}{5}$ is the only common multiplier/ factor/divisor $\frac{1}{20}$ is the GCD of the two fractions.

Think if you find the GCD of $\frac{1}{4}$ and $\frac{3}{11}$. Let us start determining the GCD by finding 10 multipliers for each.

Table 2.5

fraction	multipliers
$\frac{1}{4}$	$\frac{1}{4},\frac{1}{8},\frac{1}{12},\frac{1}{16},\frac{1}{20},\frac{1}{24},\frac{1}{28},\frac{1}{32},\frac{1}{36},\frac{1}{40}$
$\frac{3}{11}$	$\frac{3}{11},\frac{3}{22},\frac{1}{11},\frac{3}{44},\frac{3}{55},\frac{1}{22},\frac{3}{77},\frac{3}{88},\frac{1}{33},\frac{3}{110}$

Task: Determine the multipliers of $\frac{3}{11}$ as in Table 2.3.

Now tell us what the common multipliers of the two fractions will be. There are no common multipliers in the table. But surely the two fractions have one common multiplier. Now let us find 15 common multipliers of each

Table 2.6

Fraction	Multipliers
$\frac{1}{4}$	$\frac{1}{4},\frac{1}{8},\frac{1}{12},\frac{1}{16},\frac{1}{20},\frac{1}{24},\frac{1}{28},\frac{1}{32},\frac{1}{36},\frac{1}{40},\frac{1}{44},\frac{1}{48},\frac{1}{52},\frac{1}{56},\frac{1}{60}$
$\frac{3}{11}$	$\frac{3}{11},\frac{3}{22},\frac{1}{11},\frac{3}{44},\frac{3}{55},\frac{1}{22},\frac{3}{77},\frac{3}{88},\frac{1}{33},\frac{3}{110},\frac{3}{121},\frac{1}{44},\frac{3}{143},\frac{3}{154},\frac{1}{55}$

Now notice, we have found a common multiplier, which is $\frac{1}{44}$. Now we can say $\frac{1}{44}$ is the determined GCD. Now think about, number of multipliers was fixed for positive whole numbers, hence we could determine the common multipliers or the GCD like this whenever we want. But for fractions it is not fixed. Hence, we could not understand how many multipliers we would require. For example, we could find the GCD of $\frac{1}{6}$ *and* $\frac{1}{8}$ just finding 4 multipliers each. On the other hand, we needed at least 8 multipliers for $\frac{1}{4}$ *and* $\frac{2}{5}$ to determine the GCD. Again, later we have seen, for $\frac{1}{4}$ *and* $\frac{3}{11}$ it was not enough just to find 10 multipliers. We needed to find at least 12 multipliers to determine the GCD. This is very time consuming and the minimum number of multipliers is indefinite for each fraction to find the first common multiplier or the GCD.

So, let us think if we can solve this problem or not. Here, try to think if we can apply the idea of same denominator? Observe that we can make $\frac{1}{4}$ *and* $\frac{3}{11}$ into fractions with same denominators if we want to. LCM of 4 and 11 is 44. So, converting to fractions with same denominator, we get $\frac{1}{4} = \frac{11}{44}$. Because you get 44 when 4 is multiplied by 11.

Similarly, $\frac{3}{11} = \frac{3\times4}{44} = \frac{12}{44}$.

Now you can see, both the fractions have the same denominator. So, we do not need to worry about the GCD of the denominators. Now what if we multiplied both the fractions by 44? Then we would have got two whole numbers, which are 11 and 12. Now what is the GCD of 11 and 12? You know how to find the GCD of positive whole numbers. So we can say that the GCD of 11 and 12 is 1. So you see, to maintain the equality, we need to divide the GCD of 11 and 12 by 44. Since the fraction we obtained were $\frac{11}{44}$ and $\frac{12}{44}$. That means the GCD will be $\frac{1}{44}$.

What do you understand from here? If several fractions have the same denominator, that is the fractions are with same denominators, then the GCD of the fractions will be a fraction too, the denominator of which will be the GCD of the denominators of fractions, and the numerator of the fraction will be the GCD of the numerators of the fractions.

Which numerator of the two fractions with equal denominator is bigger? Of course, 12 is bigger of the two. Now think about, what was the minimum number of multipliers determined to find the GCD of the two fractions?

Task: Determine the GCD of all the fractions given earlier, by changing the fractions with equal denominators. Then determine 10 common multipliers using the GCD.

Now let us see if we can determine some more GCDs.

Suppose the two fractions are $\frac{3}{5}$ and $\frac{6}{13}$.

Task: Determine the common multipliers and the GCD of the two fractions through finding multipliers. What is the minimum number of multipliers to be determined for both the fractions to obtain the GCD?

Now let us try to change the two fractions with equal denominators. The GCD of denominators of the two fractions 5 and 13 is 65

So, the changed fractions with equal denominators will be,

$\frac{3}{5} = \frac{3\times13}{65} = \frac{39}{65}$	$\frac{6}{13} = \frac{6\times5}{65} = \frac{30}{65}$

What do we have to do now? Keeping the denominator unchanged, determine the GCD of 30 and 39. The GCD of 30 and 39 is 3

Task: Use any method and find the GCD of 30 and 39.

Hence, the GCD of the two fractions obtained is $\frac{3}{65}$.

Now the GCD of the numerators of the two fractions with equal denominators is 3. That means, if we divided the two fractions by 3, what would be the two fractions? $\frac{10}{65}$ and $\frac{13}{65}$. Which is the largest among their numerators? Earlier, when you determined the GCD using multipliers, what was the minimum number of multipliers necessary to determine the common multipliers or the GCD?

If you want, you can complete the task, step by step like this. The steps of finding the GCD of the two fractions, determined earlier, are shown below.

Table 3.1

Steps	Task	Example
1	Given fractions	$\frac{3}{5}$ and $\frac{6}{13}$
2	To change the two fractions with equal denominators	$\frac{3}{5} = \frac{39}{65}$, $\frac{6}{13} = \frac{30}{65}$
3	Take the two numerators of the fractions with same denominators	39 and 30
4	To determine the GCD of the two numbers in step 3	GCD(39, 30) = 3
5	The number obtained in step 4 is the numerator of the GCD and the denominator of the changed fractions with same denominators, is the denominator of the GCD	$\frac{3}{65}$

Observe something here! Throughout the process, we have worked with 2 fractions for example. But if you want, you can determine the GCD of more than two fractions using the whole process shown earlier.

Task: 1. Find the GCD of the following fractions, by finding the multipliers and by changing the fractions into fractions with same denominators.

i) $\frac{1}{5}$ and $\frac{3}{10}$ ii) $\frac{1}{6}$ and $\frac{5}{8}$ iii) $\frac{2}{7}$ and $\frac{6}{8}$ iv) $\frac{1}{7}$ and $\frac{1}{11}$ v) $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ vi) $\frac{1}{5}$, $\frac{3}{10}$ and $\frac{7}{15}$

2. Write down the minimum number of multipliers you needed to find for each fraction in each sum in problem 1.

3.Can you determine any relationship between the Task in number 2 after transforming the fractions with equal denominator and comparing the elements of the numerators?

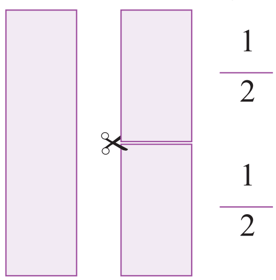
We know what the multiple of positive whole numbers is. Can you tell which are the multiples of positive whole numbers? Easily said, if a fixed positive whole number is multiplied with another positive whole number, then the product obtained is the multiple of the fixed whole number. For example, what can be the multiples of 3? They can be the infinitely many numbers like 3, 6, 9, 12, ……. Because we know the whole numbers are infinitely many. Then according to the example, it is possible to obtain infinitely many whole numbers to multiply with 3. Hence, the number of multiples of any positive whole number may also be infinitely many.

There is a relationship of the multiples with LCM. LCM in full is the Least Common Multiple. Here also several numbers are required to determine the LCM. We notice after finding multiples of several numbers that there are one or several numbers which are multiple of the numbers mentioned. That one or several multiples are called Common Multiples. The Smallest multiple among them, is called Least Common Multiple or LCM. What if there is only one multiple? It is possible to determine the LCM of fractions in this method alongside positive whole numbers.

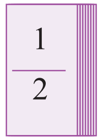
Multiples of Common Fractions

You have learnt about multipliers/factors of fractions. Now we shall try to learn about multiples of fractions. This time also we play a game. Here also we shall play a game like determining multipliers. But in a reverse way.

Look for multiples



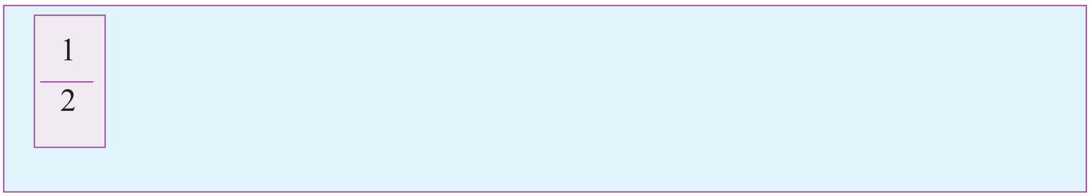
You all get divided into some groups. As before, take a piece of paper and cut it into two equal parts. Thus, the paper is divided into two pieces and each piece is $\frac{1}{2}$ of the original paper. How many such pieces did you get which are $\frac{1}{2}$ of the original paper? 2 of them. Now cutting some more papers like this, make 20 such pieces. Write $\frac{1}{2}$ on each piece to identify them.



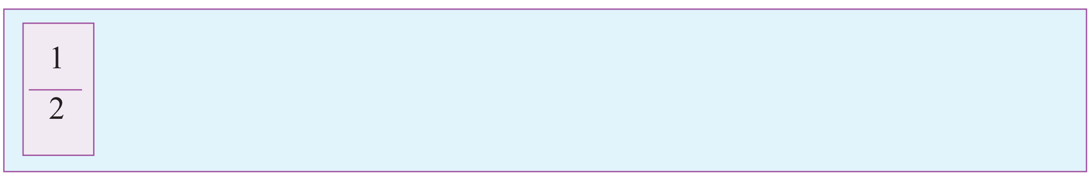
Here too, follow the same procedure of folding 3 pieces of papers into 3, 4 and 5 pieces in the game of finding multipliers.

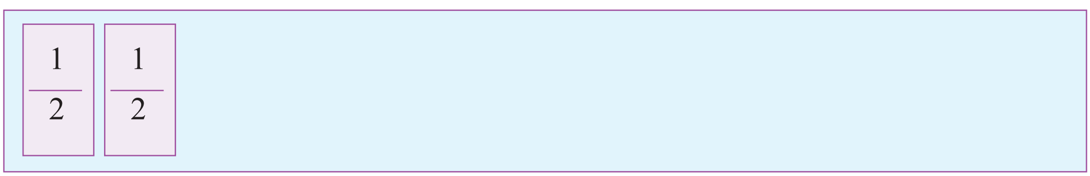
If you filled up the Table 1.1, you would notice that if you divided into 3 pieces, then each piece will be $\frac{1}{3}$. Similarly, you can find the rest. Now cutting more papers , make 20 pieces of $\frac{1}{3}$ part. Mark $\frac{1}{3}$ on each of the pieces.

Similarly, make 20 pieces for the remaining two different sizes and mark them as above. Now, first take pieces of $\frac{1}{2}$ sizes. Arrange the pieces sequentially side by side.



Place the first piece. What did you get? There remains only one piece which is $\frac{1}{2}$ part of the original paper. Now take the other piece and put it on the right side. What happened now? Two $\frac{1}{2}$ parts are set side by side. Now take one of the full paper and note that these two halves make the whole paper. Since placing two $\frac{1}{2}$ s, side by side means it is 2 times $\frac{1}{2}$. That means $\left(\frac{1}{2} \times 2\right) = 1$ part or the full part of the original paper. So, placing the $\frac{1}{2}$ pieces of the paper side by side means multiplying every time. Hence in this way put 20 pieces sequentially and fill up the following table accordingly.





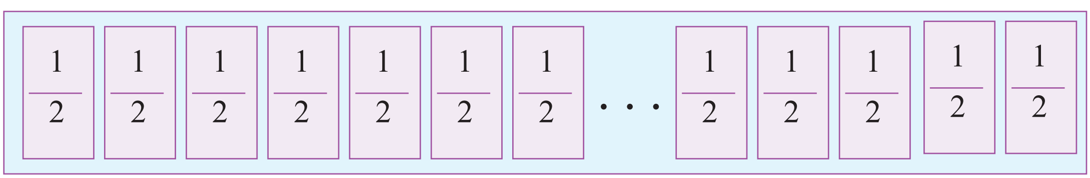


Table 4.1

(Partially filled. Complete it as your task. After that draw a table in your notebook and fill up the table for piece number 11 to 20.)

Fraction written on piece of paper	Number of pieces put side by side	Process of multiplication	Portion of original paper (in simplest form)	Number of pieces put side by side	Process of multiplication	Portion of original paper (in simplest form)
$\frac{1}{2}$	1	$\left(\frac{1}{2} \times 1\right) = \frac{1}{2}$	$\frac{1}{2}$	6		
	2	$\left(\frac{1}{2} \times 2\right) = \frac{2}{2} = 1$	1	7		
	3			8		
	4			9		
	5			10		

What do you see now? Each time you are putting a piece side by side and corresponding to that you are getting a fraction or a whole number.

Task: For the 3, 4 and 5 equal pieces of the paper, draw a table like Table 4.1 in your exercise book and complete it.

Think, what are we actually getting through this process? Start thinking about that $\frac{1}{2}$ piece from that example. Putting 3 of the $\frac{1}{2}$ pieces side by side, in fact means multiplying that by 3. That means, from the game of putting the pieces of papers, we are in fact multiplying a fraction by a whole number. Putting the pieces (of same fractions) side by side, means multiplying the fraction with the number of pieces (which is a whole number).

What is the actual output here? Remember, in case of positive whole numbers, multiplying a fixed positive whole number by another positive whole number, we obtained a multiple of that fixed positive whole number. When we are doing the same task for ordinary fractions, then those become multiples of the ordinary fractions, since the multiple or the product becomes a fraction or a whole number.

That means, multiplying a fraction by a whole number, the fraction or the whole number that we obtain, is the multiple of that fraction.

Now we prepare a table of multiples by completing the following table. Find the first 10 multiples for each fraction. The table is partially filled up.

Table 4.2

Fraction	Multiple
$\frac{1}{2}$	$\frac{1}{2}$, 1
$\frac{2}{3}$	
$\frac{1}{3}$	
$\frac{3}{4}$	
$\frac{1}{4}$	
$\frac{4}{5}$	
$\frac{1}{5}$	

If you want, you can determine the multiples of any fraction in this way.

Task: Pick 5 ordinary fractions of your choice and determine 10 multiples of each of them.

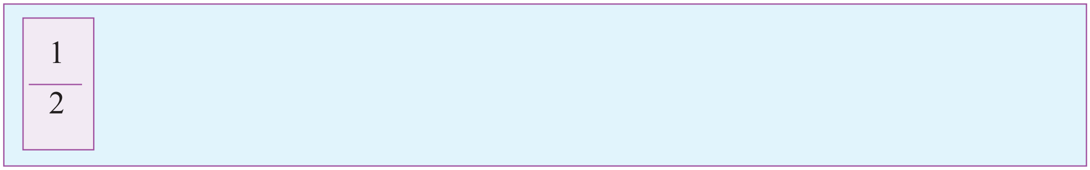
Now think, can you find all the multiples of any positive whole number? You cannot accomplish that. We have learnt earlier that the number of multiples of a positive whole number may be infinite, since the number of positive whole numbers is infinite. Similarly, you can see from above that, the number of multiples of an ordinary fraction is also infinite. It is because when an ordinary fraction is multiplied with another whole number, we always get another fraction or a whole number.

Common Multiple and LCM of Ordinary Fractions

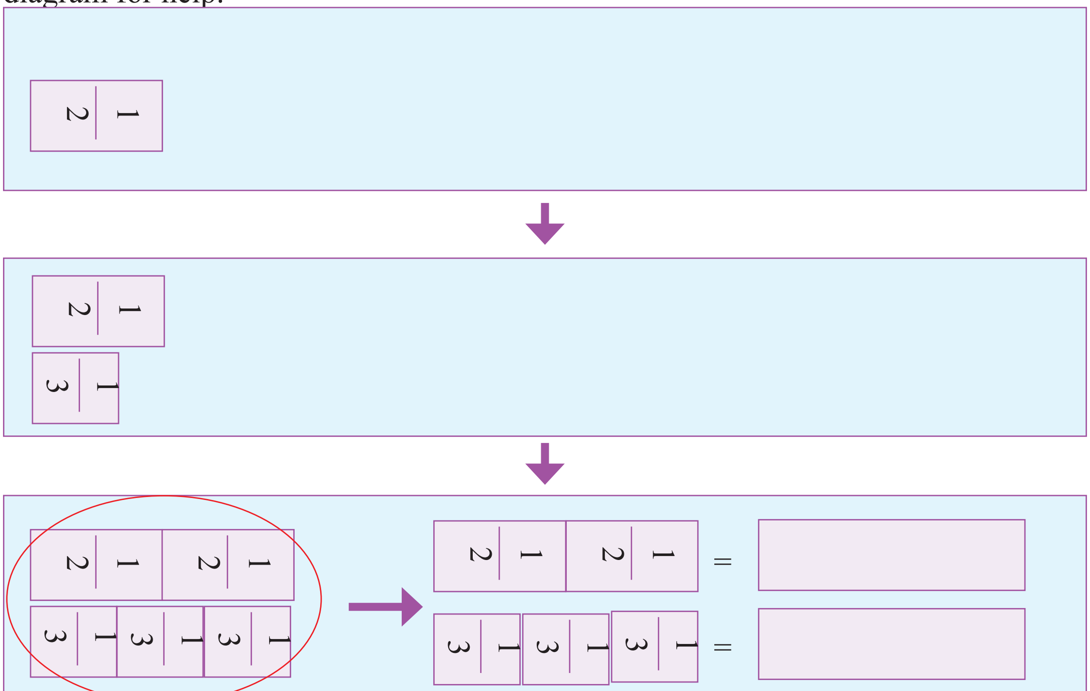
So far, we have determined what a multiple of an ordinary fraction is. Now we shall try to understand what the common multiples of ordinary fractions are. The main idea in this case is like the common multiples of positive whole numbers. That means, common multiple will be determined by comparing with the multiples of several fractions.

Let us play the game again with the pieces of papers to look for the multiples. You get divided into some groups again. We know that to determine the common multiples you need to compare several fractions. So, you need to compare the following fractions.

i) $\frac{1}{2}$ and $\frac{1}{3}$ ii) $\frac{1}{3}$ and $\frac{1}{4}$ iii) $\frac{1}{4}$ and $\frac{1}{5}$ iv) $\frac{1}{2}$ and $\frac{1}{4}$



Now each group take the pieces of papers in the game of finding multiples and again place them side by side. But since we are working with two fractions, hence we shall take two pieces of fractions. For example, what shall we do in case of $\frac{1}{2}$ and $\frac{1}{3}$? First, place a piece of $\frac{1}{2}$. Next place a piece of $\frac{1}{3}$ below that. You can use the following diagram for help.



Similarly, place another piece of $\frac{1}{2}$ next to the $\frac{1}{2}$ and another piece of $\frac{1}{3}$ next to the $\frac{1}{3}$. Continue like this. Now your task is to identify the places where the pieces are identical. Look at the diagram for help. You note that 2 of the pieces of $\frac{1}{2}$ and 3 of the pieces of $\frac{1}{3}$ are identical at one place. If you look carefully, you will note that both of them are identical at 1 whole part or equal to the whole piece of paper.

In this way, place all the 20 pieces. Identify the places where they meet together and fill up the Table. Similarly, complete the process for (ii), (iii) and (iv) and accordingly fill up the following Table.

Table 4.3

(Partially filled up. Fill up according to the directed Tasks)

Fractions	Places where the pieces meet. (Separate each piece by a comma)
$\frac{1}{2}$ and $\frac{1}{3}$	1,
$\frac{1}{3}$ and $\frac{1}{4}$	
$\frac{1}{4}$ and $\frac{1}{5}$	
$\frac{1}{2}$ and $\frac{1}{4}$	

Now let us consider another example. Now we consider again the fractions $\frac{1}{6}$ and $\frac{1}{8}$. Now we shall find 10 multiples of both of these fractions. It is shown in the following Table.

Table 4.4

Fraction	Multiples
$\frac{1}{6}$	$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}$
$\frac{1}{8}$	$\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1, \frac{9}{8}, \frac{5}{4}$

Identify in the table above, which two fractions or whole numbers exist in both rows?

Task: Determine the common multiples of the following fractions by finding 10 multiples of each.

1) $\frac{1}{3}$ and $\frac{1}{5}$ 2) $\frac{1}{5}$ and $\frac{1}{6}$ 3) $\frac{1}{3}$ and $\frac{1}{10}$

Easily you can see that $\frac{1}{2}$ and 1 are in both the rows. Hence, from the idea of common multiples of whole numbers, it is possible to say that both $\frac{1}{2}$ and 1 are common multiples of $\frac{1}{6}$ and $\frac{1}{8}$.

Now, let us get back to the previous example. We obtained the common multiples $\frac{1}{2}$ and 1 from there. Now can you tell which is the smallest of them? We thought about the matter while determining the GCD of ordinary fractions. Here it can easily be said that $\frac{1}{2}$ is the smallest of $\frac{1}{2}$ and 1. So we can say that $\frac{1}{2}$ is the smallest or Least Common Multiple or LCM of $\frac{1}{6}$ and $\frac{1}{8}$.

Task: Previously you determined the common multiples of the following fractions. Now determine the

1) $\frac{1}{3}$ and $\frac{1}{5}$ 2) $\frac{1}{5}$ and $\frac{1}{6}$ 3) $\frac{1}{3}$ and $\frac{1}{10}$

Observe that we have been working with two such fractions, whose numerators were only 1. Now let us think a little differently. Now let us consider the fractions $\frac{1}{4}$ and $\frac{2}{5}$. Determine the LCM of them.

According to the rules of determining LCM, we try to find the multiples of them. What will be the multiples of $\frac{1}{4}$? Find 10 multiples of $\frac{1}{4}$ in the Table.

Table 5.1

Fraction	Multiples
$\frac{1}{4}$	$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}$

Now imagine, what will be the multiples of $\frac{2}{5}$? Let us try to find them.

Table 5.2

Fraction	Whole no.	Multiplication process to find multiples	Simplest form of multiples	Whole numbers	Multiplication process to find multiples	Simplest form of multiples
$\frac{2}{5}$	1	$\left(\frac{2}{5}\times 1\right) = \frac{2}{5}$	$\frac{2}{5}$	6	$\left(\frac{2}{5}\times 6\right) = \frac{12}{5}$	$\frac{12}{5}$
	2	$\left(\frac{2}{5}\times 2\right) = \frac{4}{5}$	$\frac{4}{5}$	7	$\left(\frac{2}{5}\times 7\right) = \frac{14}{5}$	$\frac{14}{5}$
	3	$\left(\frac{2}{5}\times 3\right) = \frac{6}{5}$	$\frac{6}{5}$	8	$\left(\frac{2}{5}\times 8\right) = \frac{16}{5}$	$\frac{16}{5}$
	4	$\left(\frac{2}{5}\times 4\right) = \frac{8}{5}$	$\frac{8}{5}$	9	$\left(\frac{2}{5}\times 9\right) = \frac{18}{5}$	$\frac{18}{5}$
	5	$\left(\frac{2}{5}\times 5\right) = \frac{10}{5}$	2	10	$\left(\frac{2}{5}\times 10\right) = \frac{20}{5}$	4

From the tables, we can easily see that the two fractions have only one common

multiple and that is 2. Now the question is should we call this the LCM?

Now imagine, the LCM of ordinary fractions obeys almost fully, the rules of whole numbers. What have we noticed while determining the LCM of whole numbers? The first common multiple obtained among the two numbers, is the LCM. For example, what is the LCM of 6 and 8?

Multiples of 6 are: 6, 12, 18, 24, …….

Multiples of 8 are: 8, 16, 24, 32, ……

Notice here, 24 is the first common multiple of the two numbers and we can say that this is actually the LCM of 6 and 8.

Again, we determine 12 multiples for each of $\frac{1}{6}$ and $\frac{1}{8}$, which will look like the following Table 5.3

Fraction	Multiples
$\frac{1}{6}$	$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}, 2$
$\frac{1}{8}$	$\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1, \frac{9}{8}, \frac{5}{4}, \frac{11}{8}, \frac{3}{2}$

Comparing with the previous one, we are getting a new common multiple, which is $\frac{3}{2}$.

Now observe, we can find some relationship between the common multiples if we want, like the common multipliers/factors. Observe below,

$\frac{1}{2} \times 1 = \frac{1}{2}$	$\frac{1}{2} \times 2 = 1$	$\frac{1}{2} \times 3 = \frac{3}{2}$

That means, if multiples are determined sequentially like the multipliers, then using the common multiple obtained first, it is possible to find the other common multiples too. That means if one common multiple can be determined by sequentially multiplying, then the other multiples may also be determined. In that case the subsequent multiples can be determined by multiplying the common multiple obtained first, gradually multiplying by whole numbers.

Now what was happening in the game of finding multiples, when pieces of paper were being placed side by side? Each time the total part we obtained was larger than the fraction and eventually, some of the fractions were bigger than the original paper.

Now what can we decide from here? In each case we can see that multiplying the fraction by whole numbers, we were getting a bigger number compared to the original fraction. That means, multiplying the fraction by whole numbers, the new product obtained is definitely larger than the original number.

Again, sequentially multiplying the first common multiple obtained from the fractions, get the subsequent common multiples.

That means, if the multiple is determined from several fractions, the common multiple obtained first, will always be the least among them. So, the first obtained multiple is the Least Common Multiple or LCM.

Now what can we say? We determined 10 multiples for each of $\frac{1}{4}$ and $\frac{2}{5}$ and obtained only one common multiple that 2 is the LCM of the two fractions. Now imagine if you can determine the LCM of $\frac{1}{4}$ and $\frac{3}{11}$? Then find 10 multiples of each and try to determine the LCM.

Table 5.4

Fraction	Multiple
$\frac{1}{4}$	$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}$
$\frac{3}{11}$	$\frac{3}{11}, \frac{6}{11}, \frac{9}{11}, \frac{12}{11}, \frac{15}{11}, \frac{18}{11}, \frac{21}{11}, \frac{24}{11}, \frac{27}{11}, \frac{30}{11}$

Task: Find the multiples of $\frac{3}{11}$ as in Table 5.2.

Now can you say what the common multiple of the two fractions is? A common multiple cannot be obtained from the Table. But surely there will be a common multiple of the two fractions. So, let us find 15 multiples of each fraction, as in finding the GCD.

Table 5.5

Fraction	Multiple
$\frac{1}{4}$	$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3, \frac{13}{4}, \frac{7}{2}, \frac{15}{4}$
$\frac{3}{11}$	$\frac{3}{11}, \frac{6}{11}, \frac{9}{11}, \frac{12}{11}, \frac{15}{11}, \frac{18}{11}, \frac{21}{11}, \frac{24}{11}, \frac{27}{11}, \frac{30}{11}, 3, \frac{36}{11}, \frac{39}{11}, \frac{42}{11}, \frac{45}{11}$

Now observe that we have found a common multiple, which is 3. So, this 3 is the common multiple of the two fractions.

You must have noticed while finding the GCD of ordinary fractions, the problem we faced in finding the GCD, using only multipliers, we are facing the same problem while determining the LCM using only multiples.

Now the interesting matter is, the way we determine the LCM of positive whole numbers by finding multiples, the method is same for ordinary fractions. In case of positive whole numbers, it is time consuming to find the LCM like this as the number of multiples of positive whole numbers is not fixed like its multipliers. Similarly, number of multiples of ordinary fractions is not fixed and it is infinitely many.

For this reason, like we noticed before for multipliers, here also we shall not know exactly how many multiples will be needed.

For example, we could have obtained the LCM of $\frac{1}{6}$ *and* $\frac{1}{8}$ by finding 4 multiples of each. Again, we would need at least 8 multiples to find the LCM of the two fractions $\frac{1}{4}$ and $\frac{2}{5}$. On the other hand, we have seen that, finding 10 multiples were not enough to find the LCM of $\frac{1}{4}$ and $\frac{3}{11}$. The LCM can be obtained by finding at least 12 multiples. This matter is very lengthy and the minimum number of multiples to be determined for each fraction to obtain the first common multiple that is the LCM, is also not fixed.

If you have gone through the part of GCD, then you will know easily that there is an easy solution of this. We have seen earlier that if we convert the fractions $\frac{1}{4}$ and $\frac{3}{11}$ with equal denominators, we get, $\frac{1}{4} = \frac{11}{44}$ and similarly $\frac{3}{11} = \frac{12}{44}$.

Again, what have we seen about GCD earlier? Since the denominators of the two fractions are same, temporarily multiplying the fractions with the equal denominator or 44 and getting 11 and 12. Now we cannot determine the GCD as before. We have to determine the LCM. You can find the LCM of whole numbers. From that idea, LCM of 11 and 12 is 132. Now our converted two fractions are $\frac{11}{44}$ and $\frac{12}{44}$. So, to maintain the equality, we need to divide the LCM of 11 and 12 by 44. That means, the LCM will be $\frac{132}{44} = 3$.

What do you understand from here? If the denominators of several ordinary fractions are same, then the LCM of the fractions is also a fraction, whose denominator is the same as the equal denominators of the transformed fractions, and the numerator is the LCM of the numerators of the transformed fractions with same denominators. Remember that the LCM will be expressed in the simplest form.

Observe, among the two numerators of the fractions above, with same denominators, which one is bigger? Among 11 and 12, definitely12 is bigger. Now imagine, what was the minimum number of multiples needed to be determined to obtain the LCM of the two fractions?

> Task: Determine the LCM of all the fractions given earlier by using the transformed fractions with same denominators. Then use the LCM and find 10 common multiples for each.

Let us see if we can find some more LCMs.

Consider the two fractions $\frac{3}{5}$ and $\frac{6}{13}$.

> Task: Determine the common multiples and the LCM of the two fractions by finding multiples. What is the minimum number of multiples needed to obtain the LCM of both the fractions?

Let us first try to transform the two fractions into fractions with equal denominators. The two new fractions with equal denominators obtained after transforming are

$\frac{3}{5} = \frac{39}{65}$	$\frac{6}{13} = \frac{30}{65}$

Now what do we do? Keeping the denominators fixed, find the LCM of the numerators 30 and 39. The LCM of 30 and 39 is 390.

> Task: Using any method of determining the LCM, determine the LCM of 30 and 39.

So, the LCM of the two fractions will be $\frac{390}{65}$. In the simplest form, this LCM will be 6.

But we have seen before that, The GCD of these two fractions with same denominators is 3. That means, if we divided the two fractions by 3, the two fractions obtained would have been

$\frac{10}{65}$ and $\frac{13}{65}$. Which of the two numerators among them is the largest? Earlier you determined LCM using multiples. There, minimum how many multiples were needed to be taken to determine the common multiples or LCM?

If you want, you can complete the Task step by step like this. Steps of determining GCD of two fractions, obtained earlier, is shown in the following Table.

Table 6.1

Step	Task	Example
1	Given fraction	$\frac{3}{5}$ and $\frac{6}{13}$
2	To convert the two fractions with same denominator	$\frac{3}{5} = \frac{39}{65}$ and $\frac{6}{13} = \frac{30}{65}$
3	consider two numerator of fractions with same denominators	39 and 30
4	Find LCM of two numbers in step 3	LCM(39,30) = 390
5	Number obtained in step 4 is numerator of LCM and denominator of converted fraction with same denominators is denominator of LCM	$\frac{390}{65} = 6$

As in GCD of ordinary fractions, we worked with 2 fractions in the whole process of finding LCM. But all the process shown here is valid for determining LCM for fractions more than two.

Task: 1) Determine the LCM of the following fractions, by finding converting the fractions with equal denominators

i) $\frac{1}{5}$ and $\frac{3}{10}$ ii) $\frac{1}{6}$ and $\frac{6}{8}$ iii) $\frac{2}{7}$ and $\frac{6}{8}$ iv) $\frac{1}{7}$ and $\frac{1}{11}$ v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ vi) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}$

2) Write down the minimum number of multiples needed to find for each fraction in each of the problems in no. 1.

3) Can you find any relation between the Task in no. 2, comparing the elements of numerators of the converted fractions with same denominators?

GCD and LCM of Decimal Fractions

You have already learnt about the GCD and LCM of whole numbers and ordinary fractions. We have seen how to determine the GCD and LCM using the multipliers and the multiples.

Here, we shall learn how to find the GCD and LCM of decimal fractions.

First, try to remember, what is decimal fraction?

If an ordinary fraction is converted into the decimal form, then that is Decimal fraction. For example, 0.25 is a decimal fraction. The ordinary form of this fraction is $\frac{1}{4}$.

Now think, how can we get $\frac{1}{4}$ from 0.25? You have learnt this too. Still, in brief it can be said that there are two digits in 0.25, after the decimal. That means the ordinary form of this fraction will be $\frac{25}{100}$. Now we usually express the ordinary fractions in the simplest form. The simplest form of the fraction will be $\frac{25}{100} = \frac{25}{25 \times 4} = \frac{1}{4}$

GCD of decimal Fractions

Here, we shall learn two methods to determine the GCD and LCM.

First take two decimal fractions: 1.2 and 0.18. We shall try to determine the GCD of these two decimal fractions. We shall not work directly with multipliers or multiples to determine the GCD. We shall first think about how the two decimal fractions can be converted to whole numbers.

Note that, there is one digit after decimal in 1.2. So, if you multiply this decimal fraction by 10, then a whole number can be obtained. Why this will happen, that you have seen from the idea of converting a decimal fraction into an ordinary fraction. That means,

$1.2 = \frac{12}{10}$; $1.2 \times 10 = 12$

Now what shall we do for 1.8? Here, there are two digits after the decimal. Hence, we shall multiply by 100 instead of 10. Here also we shall obtain, as before,

$0.18 = \frac{18}{100}$; $0.18 \times 100 = 18$

Now, you must remember something while determining the GCD, that is, it is just not enough to convert to whole numbers. You must convert the decimal fractions to whole numbers, by multiplying each of the decimal fractions by the same number.

It is seen in the example above that, 1.2 has been multiplied by 10, but 0.18 has been multiplied by 100. Here, 10 and 100 are not same.

Now think, if 0.18 was multiplied by 10, would that be a whole number?

$0.18 \times 10 = \frac{18}{100} \times 10 = \frac{18}{10} = 1.8$. It can be seen that 1.8 is not a whole number.

Now see, if 1.2 is multiplied by 100, will that be a whole number?

$1.2 \times 100 = \frac{12}{10} \times 100 = 120$. 120 is a whole number.

Hence, to obtain the proper form to determine the GCD, both decimal fractions must be multiplied by 100 to convert to whole numbers.

Task: 1) Observe the example; both the numbers were multiplied by 100, the larger of the two numbers 10 and 100. Why was the larger number chosen?

2) Convert the following decimal fractions into appropriate whole numbers to determine the GCD.

i) 0.2 and 0.3	ii) 1 and 0.5	iii) 3 and 1.25	iv) 0.2, 0.004

Now think, multiplying by 100, we are getting two whole numbers 18 and 120. You know how to determine the GCD of whole numbers. GCD of the two numbers here is 6.

Task: Determine the GCD of 18 and 120, using any method of finding GCD.

Observe, we wanted to find the GCD of 1.2 and 0.18, not of 120 and 18. We obtained 120 and 18 by multiplying 1.2 and 0.18 by 100 respectively and determined the GCD of them. That means, the number 100 is there as an extra multiple. So, it is necessary to equalise this. Hence for equalising, we must divide the GCD found by 100. Hence the GCD will be $\frac{6}{100} = 0.06$. This was our first method.

There is another method. That is, converting the decimal fractions into ordinary fractions and determining the GCD using ordinary fractions.

Before starting the method of converting to ordinary fractions, think about it; does the first method of equalising look familiar?

Think, earlier, when we wanted to determine the GCD of ordinary fractions, we converted the fractions with same denominators. Then while determining the GCD, we separately determined the GCD of the numerators, then, divided by the denominator of the fractions with same denominators. Let us see this example.

What will 1.2 be when converted to ordinary fraction?

$1.2 = \frac{12}{10}$ (Here, our main aim is to determine the GCD in the form of decimal fraction. Hence, for the convenience of counting, we shall try to keep numbers such as 10 or 100 etc, in the denominator.)

Again, what shall we get when 0.18 converted to ordinary fraction? $0.18 = \frac{18}{100}$.

Now imagine, how we can determine the GCD of $\frac{12}{10}$ and $\frac{18}{100}$? We can find the GCD easily by converting them into fractions with equal denominators. The LCM of 10 and 100 is 100. So what will be the fractions with equal denominators?

$\frac{12}{10} = \frac{120}{100}$. On the other hand, $\frac{18}{100}$ remains unchanged.

Later, we shall determine the GCD by the method used to determine the GCD of ordinary fractions. That means, the GCD will be a fraction, whose denominator is the denominator of the two fractions with equal denominators and the numerator is the GCD of the two fractions with equal denominator.

Now remember, what we did in the first method? We multiplied both the numbers by 100 and obtained 120 and 18. Later we found the GCD of the two positive whole numbers. At last we divided by 100 again, to maintain the equality.

Do you understand now that the two methods actually indicate the same method? That means, basically the process of multiplication we followed to convert to whole numbers, that comes from the idea of converting ordinary fractions to fractions with same denominators.

Now it is possible to determine the GCD of several decimal fractions together using the process used above to find the GCD.

Task: Determine the GCD of the following decimal fractions.

1) 0.2 and 0.3	2) 1 and 0.5	3) 3 and 1.25	4) 0.2 and 0.004	5) 0.2 , 0.3 and 0.4

LCM of Decimal Fractions

The process of determining the LCM of decimal fractions is almost like determining the GCD of decimal fractions. We hope you can determine the GCD of decimal fractions.

Now let us observe how to determine the LCM of decimal fractions with an example. We shall try to determine the LCM of 1.5, 0.12 and 1.

Think here, what do you need to do? We have learnt that we can determine the GCD and LCM of decimal fractions using the same process.

Now let us convert the 3 decimal fractions into ordinary fractions and see what happens.

$1.5 = \frac{15}{10}$	$0.12 = \frac{12}{100}$	$1 = \frac{1}{1}$

Notice something- we have taken one whole number. But you know that you can express a whole number as a fraction too. Putting 1 as a denominator of the whole number, it is possible to express it as a fraction.

Now observe that, expressing the decimal fractions as ordinary fractions, the three denominators obtained are 1, 10 and 100. And you know that the LCM of 1, 10 and 100 is 100.

Then, by the process of converting to whole numbers, we need to multiply each number by 100. Results obtained after the multiplication are,

$1.5 \times 100 = 150$	$0.12 \times 100 = 12$	$1 \times 100 = 100$

Think, what is our next step? Our task is easy now. If you remember the method of finding the GCD, then you will understand that we have to determine the LCM of these 3 converted whole numbers. You know the method of determining the LCM. You can say directly from there that the LCM of 150, 12 and 100 will be 300.

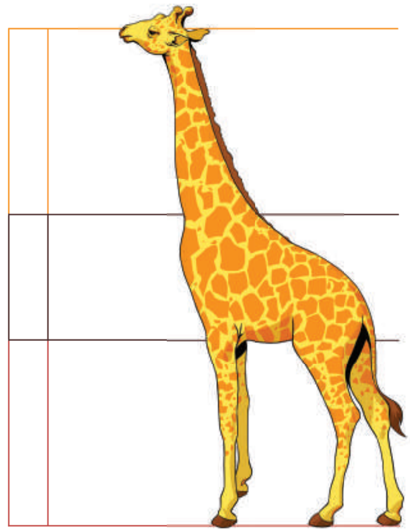
Task: Determine the LCM of 150, 12 and 100 using any method known to you.

Now tell us what do we have to do?

Now you must return to that equality process. We multiplied each number by 100. Then we found the LCM of the whole numbers we obtained. Hence we shall divide the LCM by 100. That means, the LCM of the 3 numbers is $\frac{300}{100} = 3$.

Ratio – Proportion

We obtained some ideas about **Ratio** in the previous class, and we have seen how ratio works. We shall try to learn about different types of ratios in this Chapter. Before that let us do few tasks.



Look at the animal in the picture. Do you recognise the name of the animal? This is a Giraffe. In terms of height, giraffe is the tallest animal in the animal kingdom. Now look at the giraffe. Here, you need to measure the length of the neck of the giraffe and the length of the body of the giraffe. Measure the length of the giraffe and its body along the fixed line and determine the ratio of the length of the neck and of the full height. Similarly, determine the ratio of the length of the giraffe and the length of the neck of the giraffe. Write down the two ratios you obtained in the following table:

Length of neck of giraffe	Length of full body of giraffe	Ratio of length of neck and length of full body	Ratio of length of full body and length of neck

Now pick up your Bangla and Mathematics books. Measure length and breadth and the thickness of both the books. Now determine the ratio of the lengths of Mathematics book and the Bangla book. Similarly find the ratios of the breadths and thickness too. Now fill up the following table according to the information you obtained.

	Length	Breadth	Thickness
Mathematics book			
Bangla book			
Ratio			

Simple Ratio:

So far, we have determined few ratios. Can you tell how many numbers were there in these ratios? Observe, there are 2 numbers in each ratio. If there are two numbers in a ratio, then that is called a simple ratio.

The first number in a simple ratio is called a fore number / quantity or antecedent and the second is called the after number / quantity or consequent. For example, 3:5 is a simple ratio, here 3 is the fore number / quantity or antecedent and 5 is the after number / quantity consequent.

Light Ratio

You have already measured the length of neck and the whole body of giraffe? Which was the bigger among the quantity and the quantity? You will see the fore number is smaller, the after number is bigger. These types of ratios are called light ratios.

That is, if the fore number is smaller than the after number in a simple ratio, it is called a light ratio. For example, 3:5, 4:7 etc.

The average age of students of class three in a school is 8 years and the average age of students of class five is 10 years. Here, the ratio of the ages of students of class three and class five is 8:10 or 4:5. This is a light ratio, since here the fore number is smaller than the after number.

Heavy Ratio

Let us look at the length of the giraffe again. But what can we see from the ratio of the length of the whole body and the length of the neck? This time the antecedent is bigger and the quantity is smaller. This type of ratio is heavy ratio.

That is, if the fore number is bigger than the after number in a simple ratio, then the ratio is called a heavy ratio. For example, 5:3, 7:4, 6:5 etc.

Sadia bought a packet of biscuit for 32 taka and a cone ice-cream for 25 taka. Here the ratio of the costs of the packet of biscuit and ice-cream is 32:25. Since the fore number of the ratio is 32 and the after number is 25, this is a heavy ratio.

Unit Ratio:

You have measured the ratio of two of your books. Can you tell what you have obtained there? See, what is the ratio of the length? Is not the ratio of the lengths of the two books equal or nearly equal? What can we say from the idea of ratios? Since the lengths of the two books are same, we can say the ratio is 1:1. That is both the numbers of ratio is same or unit. And this type of ratio is called unit ratio.

That is, if the antecedent and quantity of a ratio are equal, then that ratio is called unit ratio.

For example, Arif bought a ballpen for taka 15 and bought an exercise book for taka 15. Here cost of both the ballpen and exercise book are same and ratio of the casts is 15:15 or 1:1, hence unit ratio.

Task:

1) Now, figure out, what will be the types of ratios of the breadths and thicknesses of your books?

2) Now find out 1 example for each of the 3 types of ratios you learnt above from around you.

Reciprocal Ratio

Let us look at that giraffe again. Try to create a relationship between the two ratios in the following table.

Serial	Ratio	Antecedent	Quantity
1	Ratio of the length of neck and whole body		
2	Ratio of the length of whole body and neck		

What can you notice from the table? Do you find any similarity between the fore number of the 1st ratio and after number of the 2nd ratio? Again, do you find any similarity between after number of the 1st ratio and the fore number of the 2nd ratio?

Notice that one of the two ratios is reciprocal compared to the other. A ratio obtained by making the fore ratio of a simple ratio into the after ratio and the after ratio into the fore ratio is called the reciprocal ratio of the former ratio.

For example, the reciprocal ratio of 13:5 is 5:13.

Task: See if you can find any similarity between the 'reciprocal ratio' and the 'inverse fraction'.

Let us try the task of measuring our books once again. But this time include the English book together with the Bangla and Mathematics books. As before, similarly determine the lengths, breadths and the thickness of the three books and write down in the following table.

	Length	Breadth	thickness
Mathematics book			
Bangla book			
English book			

Multinumber ratio:

Think about measuring the books above, now what will you do if you are asked to find the ratio of the lengths of the books? Will you get a unit ratio as before? You will not, because now you do not have just two numbers. So, this time you have to write the three numbers side by side as a ratio.

That is, a ratio of three or more numbers is called a multinumber ratio. In this case, think about an example used before, Arif bought an eraser too for 15 taka, an exercise book for 15 taka and a ballpen for 15 taka. What will be the ratio of the costs now? Of course it will not be 15:15 or 1:1. In this case ratio of the costs will be 15:15:15 or 1:1:1. Now think, according to the example above, if Sadia bought a candy for 2 taka too, a packet of biscuit for 32 taka and a cone ice-cream for 25 taka, then what will be the ratio of the cost of these three products?

Task: What will be the ratio of the length, breadth and thickness of your three books?

Note the following information and determine the ratios accordingly.

Class	Average age
3	8
5	10
7	12

Serial	Ratio	Ratio	Simple form of ratio	Fore number	After number
1	Average age of students of classes 3 and 5	8 : 10	4 : 5	4	5
2	Average age of students of classes 5 and 7				

Serial Ratio / Continuous Ratio

In the above table, note, what is the quantity of the 1st ratio and the antecedent of the 2nd ratio? Are they not equal? Like this, if the after number of the first ratio and the fore number of the second ratio are equal to each other, then they are called serial / continuous ratio.

Think again, suppose you went to a market. Then you bought a chocolate for 10 taka, a cake for 20 taka and an ice cream for 30 taka. Can you imagine what is happening here?

The ratio of the cost of chocolate and cake you bought will be 10:20 or 1:2. Again the ratio of the cost of cake and ice cream you bought will be 20:30 or 2:3. Can we see an incident like the example we were talking about? Note, these two ratios are in a serial/continuous ratio. That is, the ratio of the cost of chocolate, cake and ice cream you bought is 1:2:3.

Task:

1. Combine the ratios of the average ages of the students of class 3, 5 and 7 given above, in a single ratio.

2. The average ages of the students of class 3 and 5 are 7 and 10 years respectively. On the other hand, the average age of the students of class 6 is 11 years. Are the average ages of these three classes in a serial ratio? If it is, what will be the serial ratio?

Individual work:

1. Fill up the following table related to ratios:

Name of Ratio	Relation	example
Simple ratio	There will be two numbers	3 : 5
Light ratio	If fore number is smaller than after number in simple ratio	5 : 8
Heavy ratio		
Unit ratio		
Reciprocal ratio		
Multinumber ratio		
Serial ratio		

2. With the help of your friend, measure the lengths from your left shoulder to your left arm and from your right shoulder to your right arm. Now measure your height. Fill up the following table with data you obtained.

Length from left shoulder to left arm (cm)	Length from write shoulder to write arm (cm)	Sum of the two previous column	Your height (cm)	Ratio of sum of lengths of shoulder to arm to your height

Can you tell what type of ratio you obtained here?

Application of Ratio in solving real life problems:

1. If you divide 500 taka among two of your friends in the ratio 2:3 who will get how much?

1^{st} friend	1^{st} friend	2^{nd} friend	2^{nd} friend	2^{nd} friend

The antecedent of the ratio is 2 and the quantity is 3. Sum of the two numbers 2+3=5.

1^{st} friend will get $\frac{2}{5}$ part of 500 taka = 500 taka $\times \frac{2}{5}$ = 200 taka

2^{nd}friend will get $\frac{3}{5}$ part of 500 taka = 500 taka $\times \frac{3}{5}$ = 300 taka

You can determine the amount of each by dividing by the sum of the antecedent and the quantity.

2. The sum of two numbers is 360. Find the two numbers if the ratio of the two numbers is 4:5. Draw a box of ratio in the following empty space.

Ratio of the two numbers is 4:5

Sum of the antecedent and quantity of the ratio = 4 + 5 = 9

The first number is = 360 of $\frac{4}{9}$ part = 360 $\times \frac{4}{9}$ = 160

The second number is = 360 of $\frac{5}{9}$ part = 360 $\times \frac{5}{9}$ = 200

The two determined numbers are 160 and 200.

3. On a Monday, the ratio of the prices of potato and brinjal per kg is 4:9, in a market nearest to you. If the price of potato is 20 taka, what is the price of brinjal?

On Tuesday, due to scarcity, if the price of brinjal increases by 5 taka per kg, what will be the new ratio?

potato	potato	potato	potato	brinjal	brinjal	brinjal	brinjal	brinjal	brinjal	brinjal	brinjal	brinjal

Antecedent of ratio is 4 and the quantity is 9. Sum of the two numbers = 4 + 9 = 13

The price of potato is 20 taka. Here the price of potato is $\frac{4}{13}$ part of total price. Price of brinjal is $\frac{9}{13}$ of total price.

Again, the ratio of the total price and the price of potato is 13:4.

Hence the total price will be $\frac{13}{4}$ part of the price of potato.

So, the total price is 20 taka $\times \frac{13}{4}$ = 65 taka.

Hence the price of brinjal will be $\frac{9}{13}$ part of 65 taka = 65 taka $\times \frac{9}{13}$ = 45 taka.

Complete the following part by drawing a box:

4. If you divide 30 oranges among three brothers Shwapon, Tapon and Monon in the ratio

5 : 3 : 2, how many will each get?

Shawpon	Shawpon	Shawpon	Shawpon	Shawpon	Tapon	Tapon	Tapon	Monon	Monon

Number of oranges = 30

Given ratio = 5 : 3 : 2 . Sum of the numbers of the ratio = 5 + 3 + 2 = 10

Shawpon gets = $\frac{5}{10}$ part of 30 oranges = 30 $\times \frac{5}{10}$ = 15 pieces

Tapon gets = $\frac{3}{10}$ part of 30 oranges = 30 $\times \frac{3}{10}$ = 9 pieces

Monon gets = $\frac{2}{10}$ part of 30 oranges = 30 $\times \frac{2}{10}$ = 6 pieces

So, Shawpon, Tapon and Monon get 15, 9, 6 pieces of oranges respectively.

Solve the following real-life problems related to ratio:

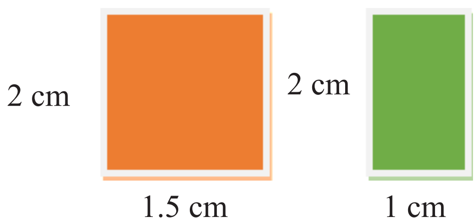
Serial	Problem	Box of ratio	Solution
1	Ratio of the ages of father and son is 14 : 3. If father is 56 years old, how old is the son?		
2	Ratio of milk and sugar in rice pudding is 5:7. If amount of sugar in that rice pudding is 4 kg, what is the amount of milk?		

3	Ratio of the cost of two books is 5:7. If the cost of the second book is 84 taka, what is the cost of the first book?		
4	Ratio of the cost of two computers is 5 : 6. If the cost of the first one is 25000 taka, what is the cost of the second one? If the cost of the first one increases by 5000 taka due to increase of price, then what is the type of their ratio?		
5	The ratio of the time of coming to school of three friends is 2:3:4. If the time taken by the first friend to go to school is 18 minutes, what are the times taken by the other two friends to go to school?		

Mixed / Compound Ratio:

You have seen that ratio is used to compare the length, width or height of two objects.

Now try to compare the two pieces of land below.



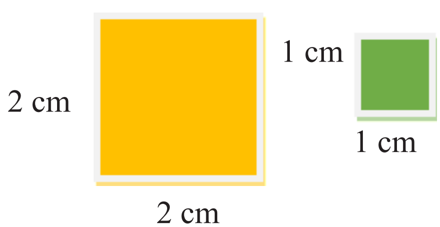
You can see that the length of the two lands is same. But the ratio of their width is $\frac{1.5}{1} = 1.5:1$.

Again, the ratio of the areas is also same $= \frac{1.5\times1}{1\times1} = \frac{1.5}{1} = 1.5 : 1$.

From this you may think, the ratio of area can be determined by the ratio of the widths.

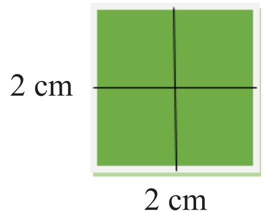
Now try to compare the two square lands below.

You need to know how many times one is bigger or smaller than the other.



The ratio of the length of the two lands $= \frac{2}{1} = 2 : 1$. If we think about this ratio, then it may appear that the 2nd square is 2 times the first square.

Look at the following diagram; is it correct to think like that?



Here, the length and width both are different. Hence, the comparison will be correct if both the ratios of length and width of the land are multiplied.

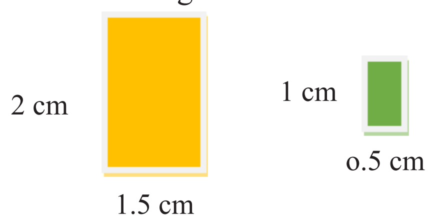
Here, the ratio of the length of the land = $\frac{2}{1}$ = 2 : 1 and the ratio of the widths of the land = $\frac{2}{1}$ = 2 : 1.

Since ratio is a fraction, multiplying the two ratios, we get $\frac{2}{1} \times \frac{2}{1} = \frac{2\times2}{1\times1} = \frac{4}{1} = 4 : 1$

Hence, we can see that it is not enough to compare just the length or just the width.

Multiplying the ratios of both the length and the width of the land, the correct ratio of the shape of the land will be obtained.

Task: Using the ratio of the length and width according to the method above, compare the sizes or the areas of the following two lands:



Why do we not determine the areas directly, and then compute their ratios! Then it is not required to calculate the ratios of the lengths and breadths separately. But in the examples above, the measurements of the lengths and breadths are given directly. So, it is possible to find the areas separately. But if only the ratios of the lengths and breadths of the two rectangles were given, then it would not have been possible to

Ratio of the lengths of two rectangular fields is 4 : 3 and the ratio of the widths is 6 : 1 What is the ratio of the areas of the field?

In this way, obtaining the antecedent of a ratio by multiplying the antecedent of several simple ratios, and the quantity obtained by multiplying the quantity of several ratios, is called a mixed / compound ratio.

For example, the mixed/compound ratio of the simple ratios 2 : 3 and 5 : 7

$= (2\times 5) : (3 \times 7) = 10 : 21.$

Example:

Determine the mixed ratio from the given simple ratios: 5 : 7, 4 : 9, 3 : 2

Solution: the product of the antecedents of the ratios = 5 $\times$ 4 $\times$ 3 = 60

And the product of the quantities = 7 $\times$ 9 $\times$ 2 = 126

Mixed ratio determined = 60 : 126 or 10 : 21

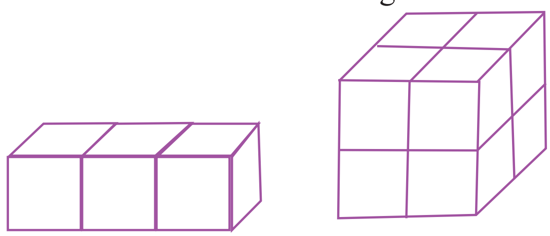
1. Determine the mixed ratio of the ratios 2 : 3, 3 : 4

2. Express the following simple ratios in a mixed ratio

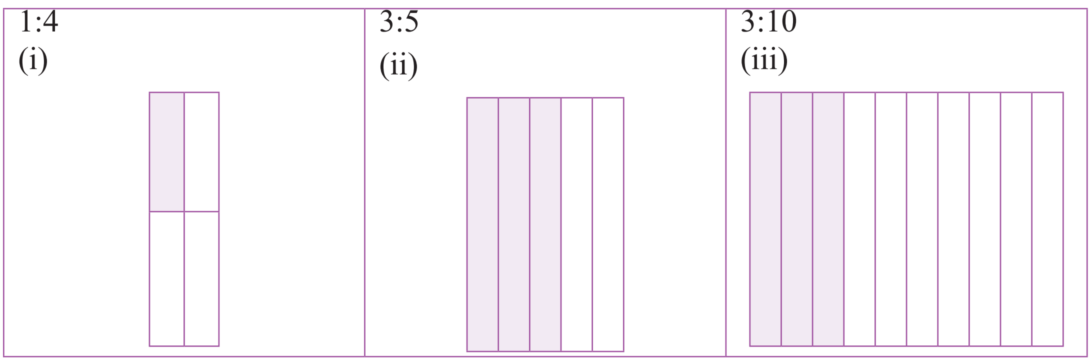
3 : 5, 5 : 7 and 7 : 9 (b) 5 : 3, 7 : 5, 9 : 7

3. All three of the length, width and the height need to be considered to compare the three-dimensional objects. That is, it is easier to compare three-dimensional objects through its volume.

Now think and see if you can find the ratio of the volume of the two rectangular solids in the following picture in some other methods than finding the volume?



Ratio and Percentages:



In the above diagram, in (a) $\frac{1}{4}$ part, in (b) $\frac{3}{5}$ part, in (c) $\frac{3}{10}$ part is ash coloured.

Here we can see,

(a) Ratio of coloured part and the full part is 1 : 4 = $\frac{1}{4} = \frac{1\times25}{4\times25} = \frac{25}{100}$ = **25%**

(b) Ratio of coloured part and the full part is 3 : 5 = $\frac{3}{5} = \frac{3\times20}{5\times20} = \frac{60}{100}$ = **60%**

(c) Ratio of coloured part and the full part is 3 : 10 = $\frac{3}{10} = \frac{3\times10}{10\times10} = \frac{30}{100}$ = **30%**

Problem: The ratio of the present ages of Jesmine and Abida is 3 : 2 and the ratio of the present ages of Abida and Anika is 5 : 1. The present age of Anika is 3 years and 6 months.

(a) Express the first ratio in percentage.
(b) What will be the age of Abida after 5 years?
(c) What is the percentage of Anika's present age compared to Jesmin's present age?

Solution:

(a) The first ratio = 3 : 2 = $\frac{3}{2} = \frac{3\times50}{2\times50} = (\frac{150}{100})\%$ = 150%

(b) Present age of Abida: present age of Anika = 5 : 1

That is, Abida's present age is 5 times Anika's present age.

Present age of Anika is = 3 years 6 months

$= (3 \times 12 + 6)$ months ($\because$ 1 year = 12 months)

$= (36 + 6)$ months

$= 42$ months

Hence Abida's present age = (42×5) months

$= 210$ months

$= \frac{210}{12}$ years (12 months = 1 year)

$= \frac{35}{2}$ years

$= 17\frac{1}{2}$ years

So, after 5 years, Abida's age will be = $(17\frac{1}{2} + 5)$ years = $22\frac{1}{2}$ years.

(c) The ratio of present ages of Jesmine and Abida = 3 : 2

That is, present age of Jesmine = $\frac{3}{2}$ times of Abida's age

From (b) Present age of Abida = $17\frac{1}{2} \times \frac{3}{2}$ years

$$= \frac{35}{2} \times \frac{3}{2} \text{ years} = \frac{105}{4} \text{ years} = 26\frac{1}{4} \text{ years.}$$

Anika's present age = $(\frac{7}{2} \div 26\frac{1}{4})$ part of Jesmine's present age

$$= (\frac{7}{2} \times \frac{14}{105}) \text{ part} = (\frac{2 \times 100}{15})\ \% = \frac{40}{3}\ \% = 13\frac{1}{3}\%$$

Hence Anika's present age is $13\frac{1}{3}$% of Jesmine's present age.

Example: Sum of two numbers is 240. If their ratio is 1 : 3, determine the two numbers. What is the percentage of the 1st number to the 2nd number?

Solution: Sum of the two numbers = 240

Their ratio = 1 : 3

Sum of the two numbers of the ratio = 1 + 3 = 4

$\therefore$ The 1st number = $\frac{1}{4}$ part of 240 = 60

$\therefore$ The 2nd number = $\frac{3}{4}$ part of 240 = 180

Again, the ratio of the two numbers = 1 : 3

$\therefore$ The 1st number is $\frac{1}{3}$ of the 2nd number = $\frac{1 \times 100}{3 \times 100} = \frac{100}{3} \times \frac{1}{100} = \frac{100}{3}\ \% = 33\frac{1}{3}\ \%$

Individual task:

The number of students in a school is 800. If at the beginning of the year 5% new students were admitted, what is the number of students at present in that school?

Problem:

Due to the reduction in price of banana by $14\frac{2}{7}$%, 10 extra banana is available for 420 taka.

(a) Determine the number if $14\frac{2}{7}$% of the number is = 10.

(b) Determine the present price of a dozen banana.

(c) How much would you have to sell a dozen banana to make a profit of 33% ?

Equivalent Ratio

Measure the picture of a school

Need to determine the width and height of your school building/structure. First measure the width and write down.

Imagine, how you can determine the height?

Now take a picture of your school building/structure and measure its width and height and write down in the table below.

Width (cm)	
Height (cm)	

Now, think, can you determine the approximate height of your school building or structure from the above?

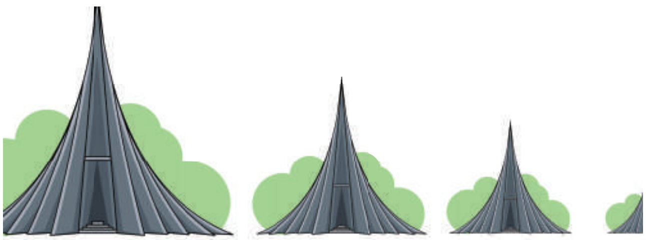
Measuring among you:

Now all of you will get divided into few groups and will measure your heights and weights. Write down the heights and weights of each one obtained in a table. Determine the heights in centimetres and weights in kilograms. Here, after your group task is complete, coordinate with the other groups and write down the information of heights and weights of all in your exercise books.

After writing the information, determine the ratio of height and weight of each one of you.

Now identify the ones whose ratios of height and weight are equal or nearly equal, and write down in bunches and divide them in groups.

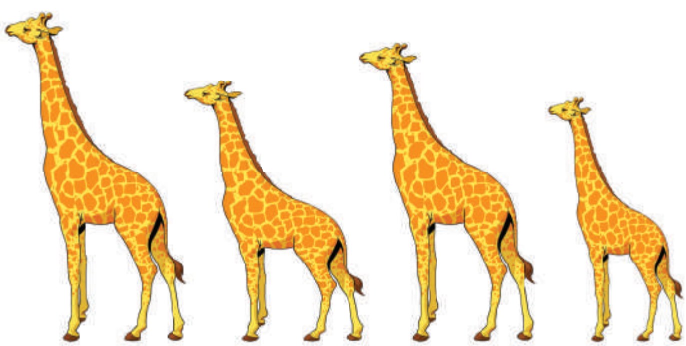
Have you seen our National Martyrs Monument? Look at the pictures of our National Martyrs Monument below.



Now measure the length and width of the pictures above, write down in the table below and find their ratios.

Picture	Height(cm)	Width(cm)	Ratio of height and width
Picture-1			
Picture-2			
Picture-3			
Picture-4			

What did you understand from the pictures? Are the ratios of the pictures same?



Can you see the giraffes below? You have measured the ratio of the length of the neck and the full body of the first giraffe. Now observe if the ratio of the length of the neck and the full body of the remaining giraffe are same or not. Measure them and fill up the following table.

Picture	Length of neck (cm)	Length of full body (cm)	ratio of length of neck and full body
Picture-1			
Picture-2			
Picture-3			
Picture-4			

What do you think of this table? Are the ratios of the giraffes equal?

Task: Now observe the following pictures, and as before, draw tables in your exercise books and determine the ratios of the height and width.

Students, let us read a Story now.

Joti and Bithi are two sisters. The like very much to play marbles. But one day after the game, they found all the marbles were lost. The next day, on the way back from school, both of them bought some marbles separately. After returning home, they found that, Joti bought 30 marbles for 50 taka. On the other hand, Bithi bought 20 marbles for 30 taka. Now find out, if both of them paid the price in the same ratio or not?

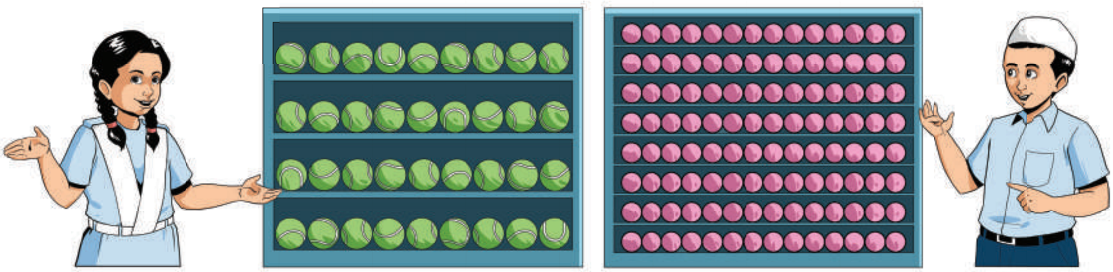
Now think, the ratio of the number of Joti's marbles and the cost is 30 : 50 or 3 : 5.

Again the ratio of the number of Bithi's marbles and the cost is 20 : 30 or 2 : 3.

So, it is observed that the ratios of both are not equal. So they did not pay in the same ratio.

Task: Among them who spent more money to buy marbles? How much should she have spent, so that she would not have spent more?

Now let us think about another story.



Mou has 36 tennis balls, on the other hand, Shubroto has 112 table tennis balls. They decided to divide the tennis balls and table tennis balls amongst themselves. So Mou gave Shubroto 18 tennis balls, on the other hand, Shubroto gave Mou 56 table tennis balls. Do you think the distribution of the tennis balls and table tennis balls was equal?

Note here, previously Mou had 36 tennis balls and she gave 18 of them to Shubroto. So the ratio of the number of balls given to Shubroto and the number of balls she initially had is

18 : 36 or 1 : 2.

Again, Shubroto had 112 table tennis balls and he gave 56 of them to Mou. So the ratio of the number of balls he gave to Mou and the number of balls he gave to Mou is 56 : 112 or 1 : 2.

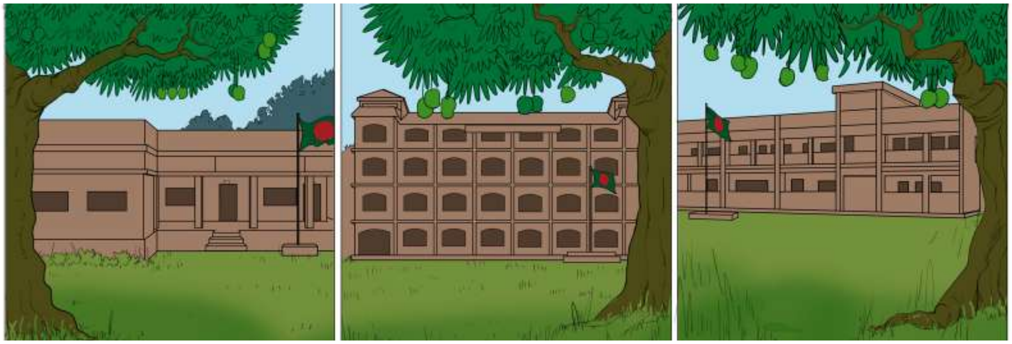
Here we need to think about something else. Let us try to find the ratio of the tennis balls and table tennis balls Mou has after sharing. That is 18 : 56 or 9 : 28.

Again, in case of Shubroto, this ratio is 18 : 56 or 9 : 28.

Since both shared the items amongst themselves in the same ratio and it is seen that after sharing, the ratios of the balls they have are equal. So, we can say that the tennis balls and table tennis balls are equally divided among the two.

Task: It is seen above that 18 tennis balls given by Mou and 56 table tennis balls given by Shubroto has equally been distributed. Think about if Mou and Shubroto could have exchanged the tennis balls and table tennis balls in a different way to divide among them equally.

In three schools, mangoes are distributed among the students after they are harvested in the following way.



In 1st School:

Class	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Number of students	72	77	74	73	70	67	66	69	75	71
Give number of mangoes	144	154	148	146	140	134	132	138	150	142

In 2nd School:

Class	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Number of students	64	61	55	56	49	58	57	62	53	50
Given umber of mangoes	192	183	165	168	147	174	171	186	159	150

In 3rd School:

Class	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Number of students	41	44	45	47	48	37	39	42	40	43
Given number of mangoes	80	90	90	95	100	75	80	85	80	86

Answer the following questions with respect to the above:

Question	1st school	2nd school	3rd school
Is there equal distribution of mangoes among students of class 1?			

If the distribution of mangoes is equal in each class, then in what ratio of students and number of mangoes has the distribution done in each class?			

Rearrange the class wise distribution of the mangoes among the students of the 3rd school such that the students of the 2nd and 3rd school get equal mangoes, class wise and fill up the table below:

class	1st School		3rd School	
	Number of students	Number of given mangoes	Number of students	Number of given mangoes
1st	72	144	41	
2nd	77	154	44	
3rd	74	148	45	
4th	73	146	47	
5th	70	140	48	
6th	67	134	37	
7th	66	132	39	
8th	69	138	42	
9th	75	150	40	
10th	71	142	43	

Keeping the information of the mangoes obtained by the students of the 2nd school, rearrange the class wise mangoes obtained by students of 1st and 3rd schools in a way that the class wise students of 1st and 3rd schools get equal number of mangoes.

Class	1st School		2nd School		3rd School	
	Number of students	Number of mangoes given	Number of students	Number of mangoes given	Number of students	Number of mangoes given
1st	72		64	192	41	
2nd	77		61	183	44	
3rd	74		55	165	45	
4th	73		56	168	47	
5th	70		49	147	48	
6th	67		58	174	37	
7th	66		57	171	39	

8th	69		62	186	42	
9th	75		53	159	40	
10th	71		50	150	43	

Making our National Flag

Dear Students, let us now know about our National Flag and do an interesting work. You all are familiar with the flag of Bangladesh. The size/shape of the flag of Bangladesh used for different places is partially given below. Now try to fill up the blanks.

No.	Places	Length	Width	Radius of red circle ($\frac{1}{5}th$ of length)
1	On buildings of different sizes	10 feet	6 feet	2 feet
2		5 feet	3 feet	1 foot
3		2.5 feet	1.5 feet	
4	In big cars		9 inches	3 inches
5	In medium/small cars and for use on tables in International or bilateral meetings	10 inches		

This is basically the ratio of the length and breadth of our national flag. Now imagine, where will be the centre of the red circle?

The rule in this case is that, from the left side of the flag, taking 9 parts of 20 parts or $\frac{9}{20}$ part of the total length of the flag, you need to draw a vertical line or a line parallel along the width. Then taking half or $\frac{1}{2}$ part of the width of the flag, need to draw a horizontal line or a line parallel along the length. The point of intersection of these two lines is the centre of the circle. The circle has to be drawn taking this point as the centre.

Now using the completed table above, fill up the following table. For your advantage, one row of the table has been filled up.

No	Place	length	Width	Radius of the red circle ($\frac{1}{5}$ of length)	Distance of 9 part out of 20 of total length from left (a line to be drawn from this point along the width)	half the distance from any side of width (a line to be drawn from this point along the length)
1	On buildings of different sizes					
2		5 feet	3 feet	1 feet	$(5 \times \frac{9}{20} = \frac{9}{4})$ = 2.25 feet	$3 \times \frac{1}{2} = \frac{3}{2}$ = 1.5 feet
3						
4	In big cars					
5	Medium/small cars or for use on tables in international or bilateral meetings					

Team Task: Get divided into three groups and according to numbers 3, 4, 5 cut out rectangular sizes of papers like the national flag. Then determine the centre of the red circle of the flag. Then draw the circle with the required radius. Then complete the task of making the flags using the required colours.

Now put the three flags you made, side by side on a table/bench. What do think of the flags? Do the shapes of all looks same?

Now using the ratios of length and width of flags obtained from the table above, write down in your exercise books as fixed ratio. Then see if the ratios of length and width are same.

What have we learnt from the tasks?

If two or more ratios are equal, then those equal ratios are called proportional/ equivalent ratios.

If two ratios are equal, that is if they are in equivalent form, the 1st and the 4th terms of the two ratios are called marginal terms and the 2nd and 3rdterms are called middle terms. That is the fore term of the 1st ratio and the after term of the 2nd ratio are called marginal terms and the after term of the 1st ratio and the fore term of the 2nd ratio are called the middle term.

Equivalent ratios are expressed with the symbol :: or symbol $\propto$

For example, consider there are two ratios 12 : 16 and 45 : 60.

In the simplest forms, the two ratios are $\frac{12}{16} = \frac{3}{4}$ and $\frac{45}{60} = \frac{3}{4}$

So, we can say these two quantities are in equivalent ratios.

This can be written as 12 : 16 :: 45 : 60 or $\frac{12}{16} \propto \frac{45}{60}$

Ratio of three numbers

The approximate distance from Dhaka to Chittagong is 250 km. A bus starting at 9.00 am from Dhaka reaches Chottogram at 2.00 pm. A table is given below showing the distance covered by the bus per hour from Dhaka. Note that the distance covered by the bus per hour is proportional to time. Look at the Table.

Time (hrs)	1	2	3	4	5
Distance(km)	50		150		250

Now watch, the distance covered by the bus at the end of 2 hours, is unknown to us. It may be anything between 50 and 150 km. But look above, it is mentioned that the distance covered by the bus per hour is proportional to time. That means if the ratio of time and distance is taken it will be same every hour. Now we see, the bus travels 50 km in the first hour. That is the ratio of time and distance is 1 : 50. Now we want to determine the distance covered at the end of 2 hours. Suppose the distance covered at the end of 2nd hour is □. Then the ratio will be 2 : □. Now watch, it said the ratios are proportional. That means equal !

We can say 1 : 50 = 2 : □

AS fractions from here we get $\frac{1}{50} = \frac{2}{\square}$

From here we get, $1 \times \square = 2 \times 50$

Hence □ = 100.

Now watch, what have we done when we multiplied in the form $1\times \square = 2 \times 50$? Look below. If we assume that a : b and c : d are proportional, then we can say that a : b = c : d.

In fractions we get, $\frac{\square}{\square} = \frac{\square}{\square}$. And by multiplying like the previous example, get$a \times \square = \square \times \square$.

Now we have learnt from proportion/equivalent ratio, that in this proportion, *a* is the 1st number, *b* is 2nd number, *c* is 3rd number and *d* is 4th number.

So, in any proportion , 1st number $\times$ 4th number = 2nd number $\times$ 3rd number.

Now look, while determining the distance travelled by the bus at the end of 2 hours, we knew the values of the 3 numbers, except the 4th number. Then using those three numbers we computed the 4th number.

Now observe the following. You are told that in some proportion 1st ,3rd and 4th numbers are 14, 7, and 22. Determine the 2nd number.

Suppose the second term is $\square$. Then from the rules learnt above, the proportion is

$14 : \square = 7 : 22$.

That is $14 \times 22 = 7\square$

Or $\square = \frac{14 \times 22}{7} = 44$

Hence the 2nd number in this proportion is 44.

So what did you understand from this? If three of the numbers of 1st number, 2nd number, 3rd number and 4th number are known in a proportion, then you can determine the unknown number.

> In this way, if the three numbers in a proportion are known, then the method to determine the unknown number is called Triple / Triplet number method.

Task: 1) Determine the distance travelled by the bus in the table at the end of 4th hour.

2) In a proportion, if the 1st , 2nd , and 4th numbers are 9, 18 and 20 respectively, what is the 3rd number?

3) Rana has 4 pencils and 5 pens. On the other hand, Shajib has 10 pens. Now if the ratios of pencils and pens of Rana and Shajib are proportional, then how many pencils does Shajib has?

4) Few cars took part in a car race of 20 km long. Among them, the information of the distance travelled by the winning car is given in fixed intervals, for up to 10 minutes.

Here, the interesting fact is that thecar raced with a fixed speed always. Now you look at the following partially fulfilled table and complete it using the idea of proportion.

Time (minutes)	1	2	3	4	5	6		8		10
Distance travelled (km)	2	4				12	14	16	18	

Serial/Ordinal /Successive Ratio:

Now let us have a look at the table of racing car above. Note that here, the car travels 2 km at the end of 1st minute and 4 km at the end of 2nd kilometre. Now think, what are the two ratios with respect to minute and distance travelled?

The ratio for the 1st minute is 1 : 2 and the ratio in the 2nd minute is 2 : 4. Now note here, the two middle terms are same, which is 2. So we get a serial/succession. This type of ratios/proportions are called serial/ordinal/successive ratio.

> The proportion in which the two middle terms of the ratios are equal, that proportion is called serial/ordinal ratios.

Now look at the example below. Mishu, Adittya and Shoborna arranged a competition of throwing marbles. There, the marbles thrown by them crossed distances of 35, 28 and 43 metres respectively. Now the ratio of the distances crossed by marbles of Mishu and Shoborna is 35 : 43. Again the ratio of the distances crossed by marbles of Shoborna and Adittya is 43 : 28. That means, from these three numbers we can take the two ratios 35 : 43 and 43 : 28. Here, the types of ratios like 35 : 43 : : 43 : 28 are called serial/ordinal ratios. The three crossed distances 35, 43 and 28 metres are called serial proportional numbers.

If you watch a bit more, you will find if a, b, c are serial proportional numbers then $\frac{\square}{\square} = \frac{\square}{\square}$ or $\square \times \square = (\square)^2$.

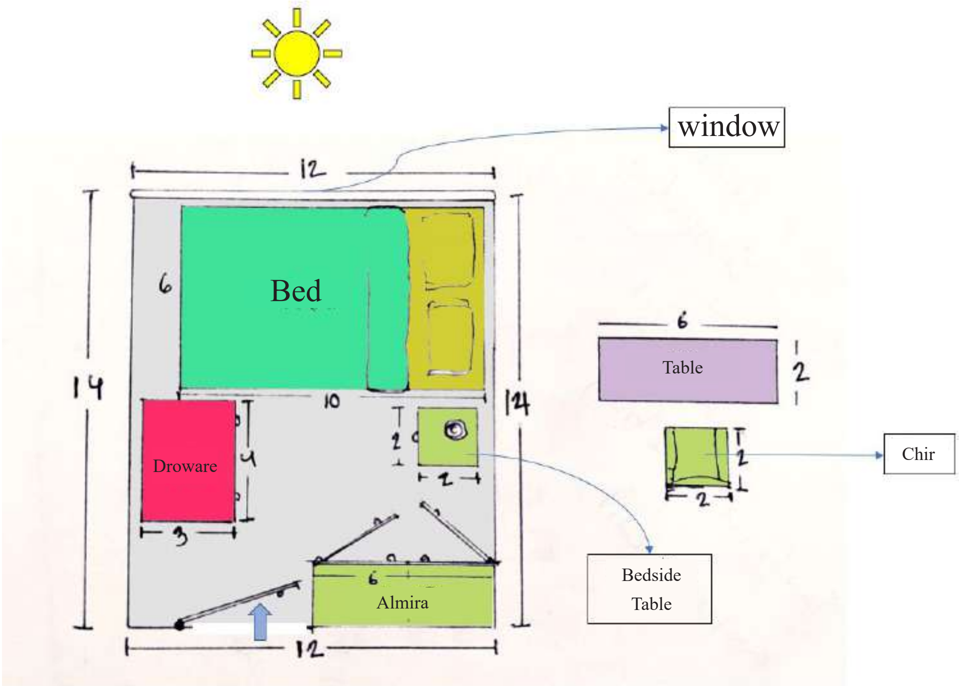
> In case of serial proportion, the product of 1st and 3rd numbers is equal to the square of the 2nd number and the 2nd term is called the middle term of the 1st and 3rd term.

Activity:

The 1st and 3rd terms in a serial proportion are 4 and 16. Determine the middle term and the serial proportion.

Geometric Shapes

Suppose your family shifted to a new home. You get a new room of your own. You have a bed, a cupboard, a drawer and a bedside table. There is also a large window which provides plenty of daylight. But there is no study table and chair for you. You got a nice room but no place to study. Isn't that disappointing? Notice the figure below, all measurements are fitted without the study table and chair. Your wish is to have sunlight on your study table during the day. Again, the cupboard is fixed with the wall, so you cannot remove that. Everything in the room is required for you, so you cannot put anything out of the room. But you can rearrange some furniture. So how can you find suitable place for your study table and chair? Let me give you a hint, you can cut papers of proportional size and try to solve the problem.



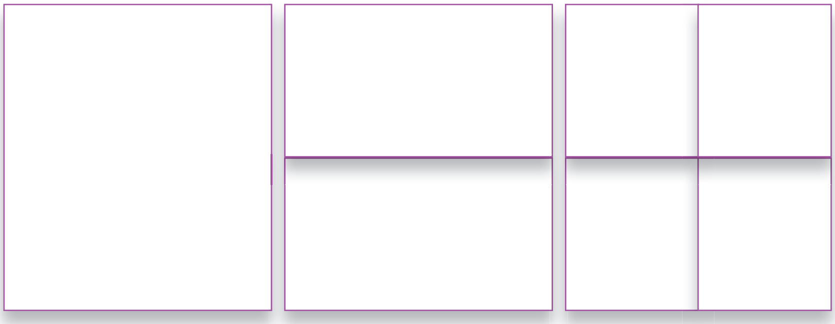
Could you find a solution? It is alright if you cannot do it now. Keep thinking about the problem and try to solve it later. Notice that, this is a problem of geometric shapes. We face many problems of this

kind daily. Solving them is easy if we have a clear concept of geometric shapes. If you follow the instructions in this part of the book and complete the team and individual works, you will build a nice concept about geometric shapes. We will solve various problems by drawing figures and cutting, folding papers. So let's start.

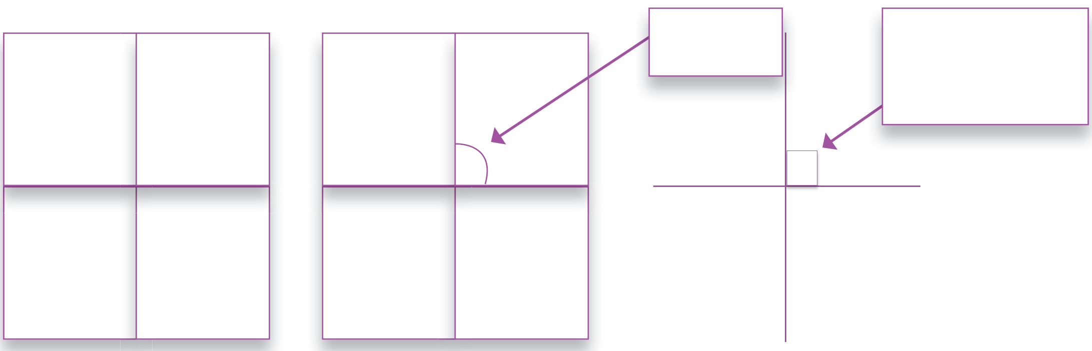
Construction of a geometric shape

We can construct various geometric shapes by properly folding papers. At first let's take a paper sheet.

Task 1: Take an A4 size paper and fold it in the middle. Fold it again transversely in the middle (like the figure).

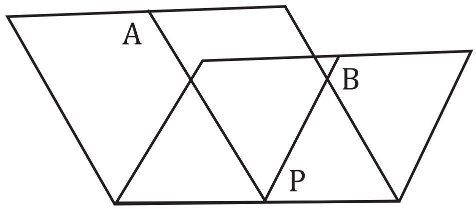


Draw a line along each fold. At the intersecting points four angles are created. Measure each of them. All are equal to the others. As we found them by folding equally, we call them right angle.



What happens if the folds are not equal? The angles are not equal too. That means, we will not get right angles. If two lines intersect at a right angle we say that the lines are perpendicular to each other.

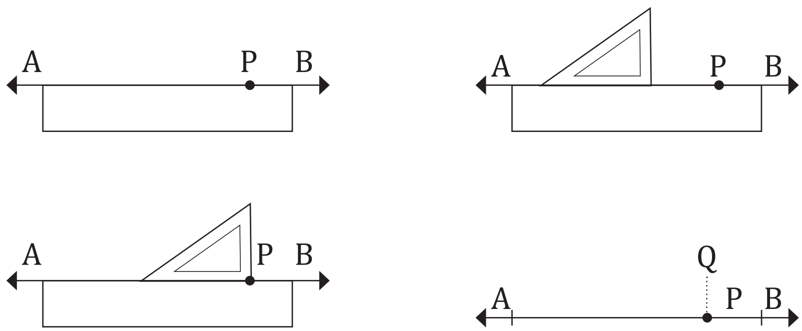
Task 2: Suppose you have a line segment AB. We want to draw a perpendicular at a point P on AB. Fold the paper so that a segment of the given line AB is folded over onto itself and so that the crease passes through the given point .



Draw a line along the fold. Now measure the angle created between the new line and AB.

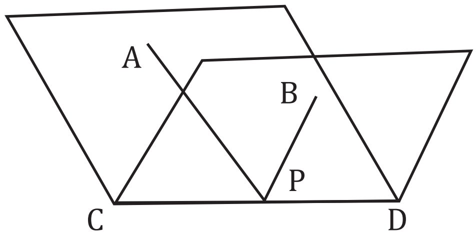
What happens if AB is not folded over onto itself? What will be the measure of the angles? Examine and discuss with your classmates.

We can also draw perpendiculars using set squares of geometry box. At first consider a point P on the line AB. Now place a side of a ruler along the line AB and hold it. Now place a side of set-square along the ruler in such a way that the right-angled vertex of the set-square coincides with the point P. Hold the set-square and draw a line PQ. Note that, there is already a right angle in the set-square, so we are easily drawing a right angle same to the given one.



Task 3: Suppose you are given a line segment AB and you need to draw another line segment equal to AB. You may think at first to measure the length using a scale and draw a line segment of the same length. But it is tough to measure when the length has fractions. So we will try another method. Take a piece of thread. Now place one end of the thread at an endpoint of the line segment and keep it straight along the line segment. Cut the thread where it meets the other endpoint. Now setting the thread straight mark the endpoints on a paper. If you join the points using a ruler, you will get the required line segment equal to AB.

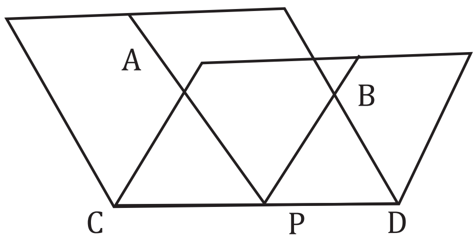
Task 4: Suppose you are given a line segment AB drawn on a paper. Fold over the point A and hold it coincident with the point B of the other portion. While these points are held tightly together by the thumb and forefinger of one hand, crease the fold with the thumb and forefinger of the other hand. Then extend the crease in both directions to form a straight line. If you measure, you can see that from any point on the crease the distances of A and B are equal. That means we divided the line segment AB into two equal parts.



You should check what happens if A and B are not coincident when we fold. How does the distance of A and B change from the crease? Note down your opinion.

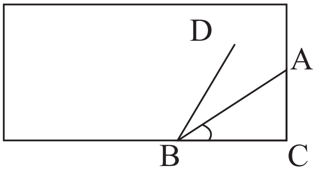
Measure the angle between AB and CD. According to the angle between them, can we say that AB and CD are perpendicular to each other? In that case we will say that CD is the perpendicular bisector of AB.

Task 5: Suppose, we are given a line AB. You are asked to draw a perpendicular on AB from a point P outside AB. From task 1 we know how to get a perpendicular. From task 4 we know how to get a perpendicular bisector. So we have to fold the paper in such a way that one part of AB coincides with another part. But we have a specific point P. So our crease should pass through the point P after folding.



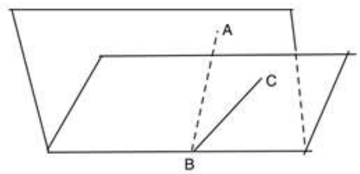
If we draw a line CD on the crease, we can see that it passes through the point P. Measure the angles between AB and CD and note them down. You can see that all of them are right angles.

Task 6: We take an angle ABC on a paper whose vertex is at B. Now we fold the paper along AB. We draw a line on the paper where the side BC falls.



Now measure and compare the angles ABC and ABD. You can see that they are equal. In this way we can draw a new angle equal to a given angle.

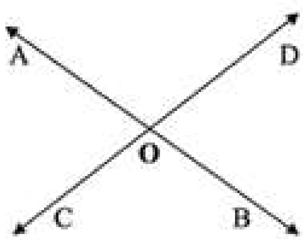
Task 7: We take an angle ABC on a paper whose vertex is at B. Now we fold the paper in such a way that the sides AB and BC coincide with each other. Notice that the crease passes through the vertex B. Now draw a line along the crease and you will get two angles.



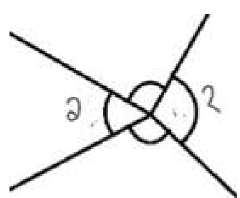
Measure the angles. Notice that they are equal to each other, and each is half of the angle ABC. We can also say this using work 6. In work 6 we took the side of the new angle along the other side of old angle. Again, notice that the distance of the crease here is in equal distance from two sides of the angle ABC. So, if we can draw a line equidistant from two sides of the angle, we will get the bisector of the angle.

Team task: Divide the class with 4-5 students in each team. Discuss and find one similarity and one difference between bisector of an angle and perpendicular bisector of a line segment.

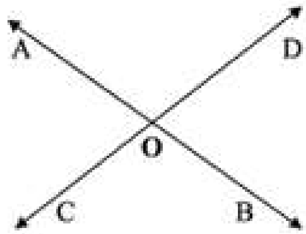
Notice the intersecting point of the pair of lines in the figure. AB and CD are two straight lines who intersect at point O. So, there are two pairs of opposite faced angles. Each pair is called vertically opposite angles. All of them have O as their vertex.



A question may arise in your mind why we are calling them vertically opposite instead of opposite angles as opposite is an easier name? Now notice the next figure. The angles marked as 1 and 2 has the same vertex. If we extend the sides of angle 1 in the opposite direction, we do not get the angle 2. Here they are only opposite angles. We will say them vertically opposite if we get one angle by extending the sides of the other in the opposite direction.



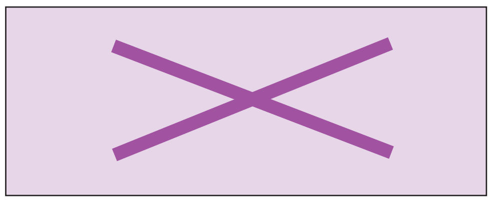
Task 8: Now take a paper on which there are two lines AB and CD who intersect at O. Now fold along the point O in such a way that the parts BO and CO coincides with each other. Notice the positions of AO and DO. Do they coincide too? What decision can be made about vertically opposite angles from here?



So we can say that if two lines intersect each other the vertically opposite angles are equal to each other.

Game of three sticks

Take two sticks. If you do not have sticks, you may take two pens/pencils. If you place them in different ways, you will observe that they can intersect at only one point. From your previous class you already know that if they do not intersect then they are parallel to each other.



Now we try to place a third stick. If the first two sticks are parallel to each other, then there are two ways of placing the third stick.

1. Third stick will be parallel to the first two (Figure 1) or,

2. Third stick will not be parallel to any of them and it will intersect both (Figure 2).

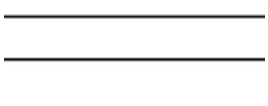


Figure 1

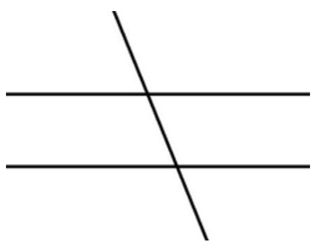


Figure 2

If the first two sticks are not parallel to each other, then there are two ways of placing the third stick.

3. Third stick will intersect the first two at their intersecting point.

4. Third stick will intersect the first two at two different points.

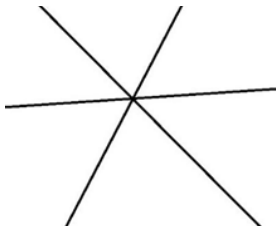


Figure 3

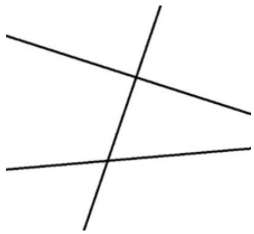


Figure 4

If more than two lines intersect each other at the same point like in the Figure 3, we say that the lines are concurrent. If one line intersects two or more lines, then the intersecting line is called a transversal like in Figure 2 and 4.

Notice Figure 5 and 6 below. If a line intersects with two other lines in two different points, eight angles are formed.

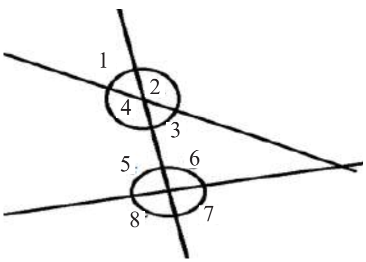


Figure 5

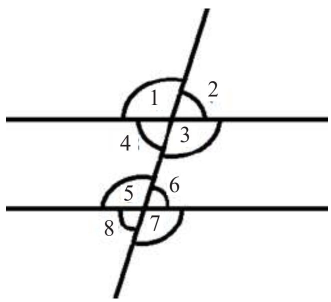


Figure 6

You will observe that 1 and 5, 2 and 6, 3 and 7, 4 and 8 these pairs are in similar places. They are above or below the pair of lines or they are at the same side of the transversal. For example, 1 and 5 are above the lines and at the left side of the transversal. These types of pairs are called corresponding angles.

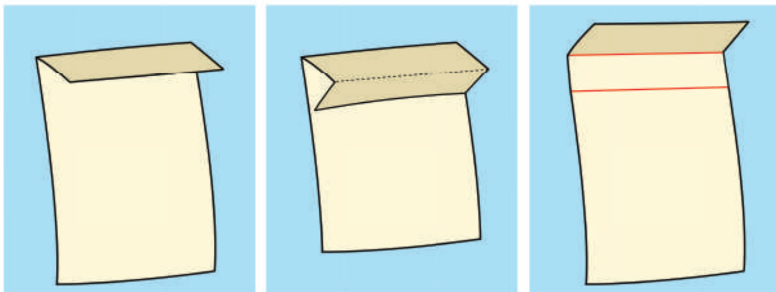
Again, you can notice that positions of 1 and 7, 2 and 8, 3 and 7, 4 and 8 are somewhat opposite. For example, 6 is at up and right and 4 is at left and below. These types of pairs are called alternate angles.

Notice that, 1, 2, 7 and 8 are outside and 3, 4, 5 and 6 are inside. They are called exterior angles and interior angles, respectively.

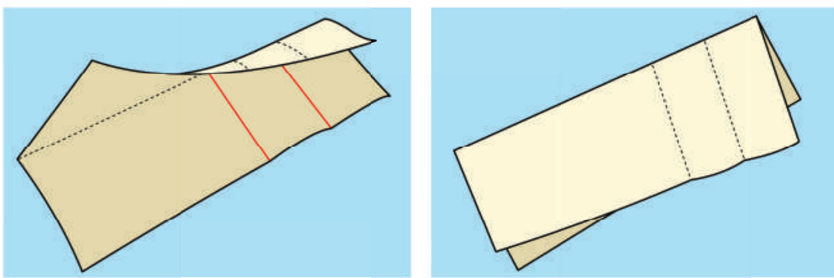
Now we will find out relationships among them.

Team work: Form a team of 4-5 students. Each team will take a paper. Now follow the steps.

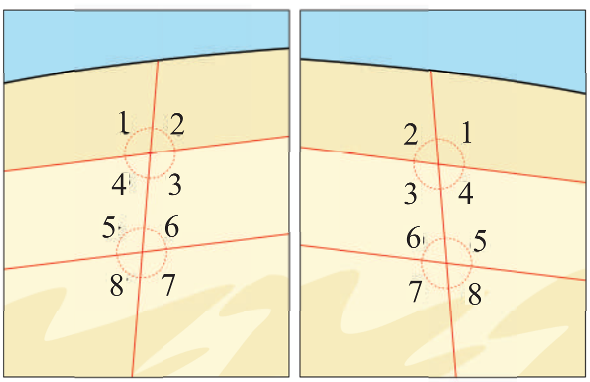
1. Fold the paper twice in the same way as the figure such that the folds are parallel. If needed ask your teacher for help. Draw a line in each crease. You will get a pair of parallel lines.



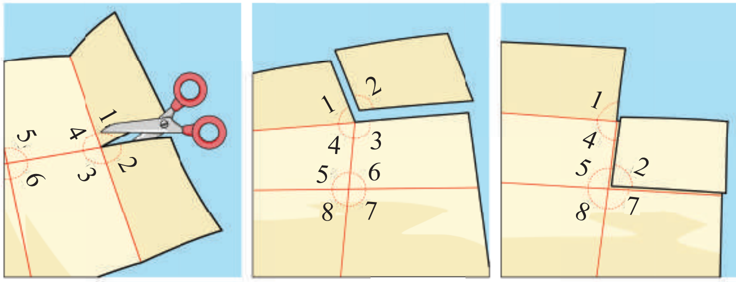
2. Like the figure fold the paper transversely near the middle part. Draw a line along the crease and you will get a transversal.



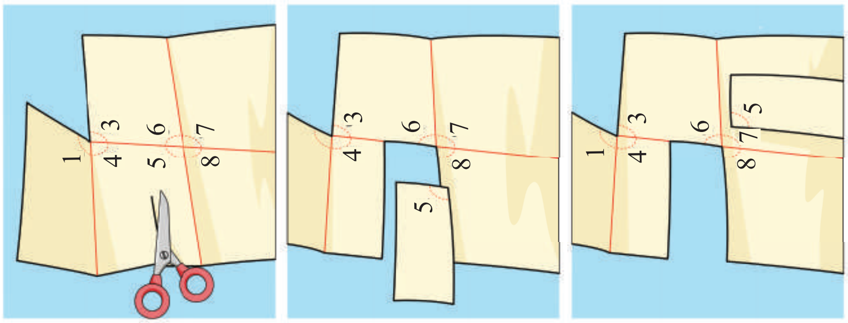
3. Draw line along the crease in both side of the paper and mark the angles using numbers like the given figure. Write the angles in such a way that in the opposite side of angle 1, the other angle 1 is also situated.



4. Now cut the number 2 angle in the same way as the figure. Match angle 2 with angle 6. You can see that they are equal. It means **when a line cuts a pair of parallel lines the corresponding angles are equal.** Can you check yourself if angel 2 can be placed on any other angle?

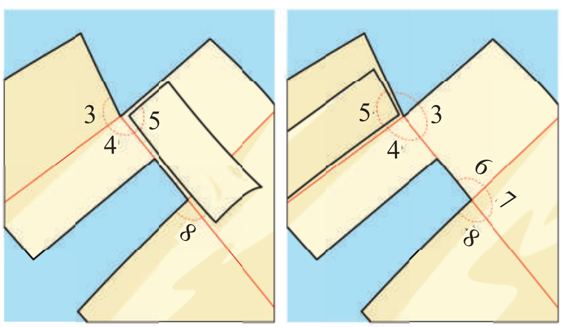


5. Now separate the angle 5 in the same way showed in the figure.



6. After placing you can see that the angle 5 matches with angle 3. We know that they are alternate angles. So, **when a line cuts a pair of parallel lines the alternate angles are equal.**

Again, angle 1 and angle 5 are corresponding, hence they must be equal. Note that, the sum of angle 4 and 5 is equal to a straight angle or two right angles. They are interior angles on the same side of the transversal. So we can say that **when a line cuts a pair of parallel lines the sum of two interior angles on the same side of the transversal is equal to two right angles.**



From this team work we learnt the following information which can be used later for solving other problems.

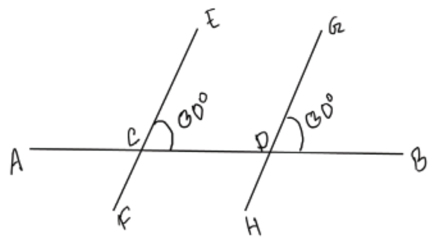
1. When a line cuts a pair of parallel lines the corresponding angles are equal. Here 1 and 5, 2 and 6, 3 and 7, 4 and 8 are corresponding angles and equal.

2. When a line cuts a pair of parallel lines the alternate angles are equal. Here 3 and 5, 4 and 6 are alternate angles and equal.

3. When a line cuts a pair of parallel lines the sum of two interior angles on the same side of the transversal is equal to two right angles. Here sum of 3 and 6, sum of 4 and 5 each is equal to two right angles.

Let's examine the pair of lines are parallel or not, if the angles around transversal show the same behavior.

Individual task: At two points of a straight line AB, draw two 50^0 angles like the figure below. Now measure the distance between the lines EF and GH.



From the results we can have the following decisions:

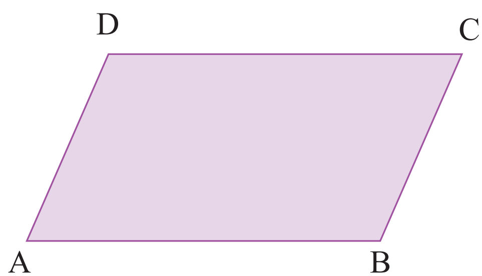
1. When a straight line cuts any pair of straight lines, if corresponding angles are equal, then the pair of straight lines are parallel.

2. When a straight line cuts any pair of straight lines, if alternate angles are equal, then the pair of straight two lines are parallel.

3. When a straight line cuts any pair of straight lines, if sum of internal angles on the same side of transversal is equal to two right angles, then the pair of straight lines are parallel.

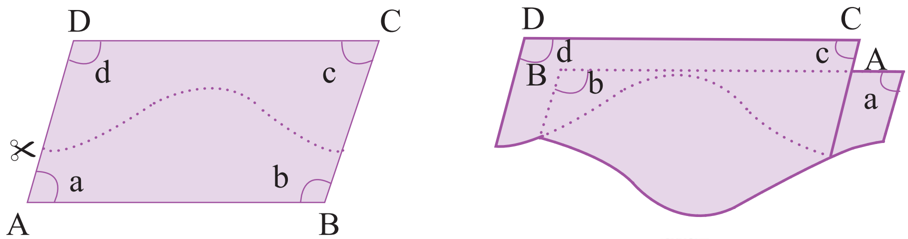
Individual Task

1. Construct some parallelograms by cutting papers.



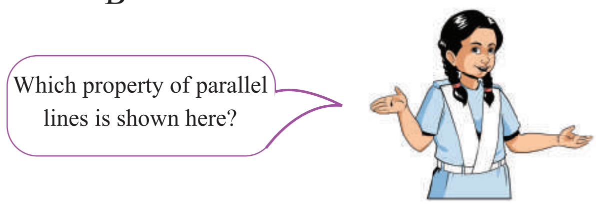
Now do the following works:

a) Cut the parallelogram into two pieces and match the angles like the given figure.

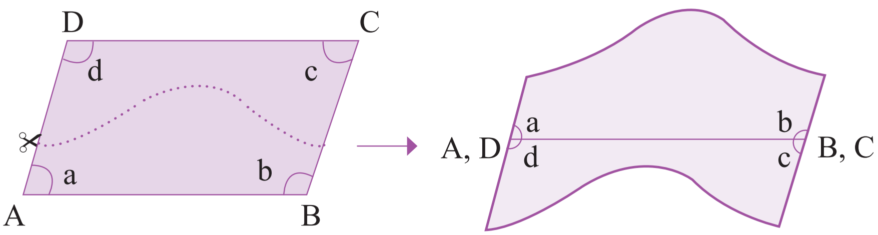


$\angle a = \angle c$

$\angle b = \angle d$

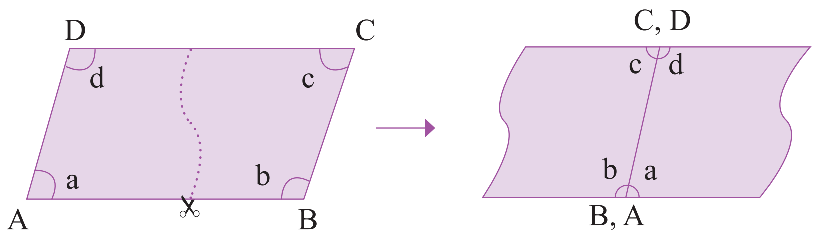


b) Cut the parallelogram into two pieces and match the opposite angles like the given figure.



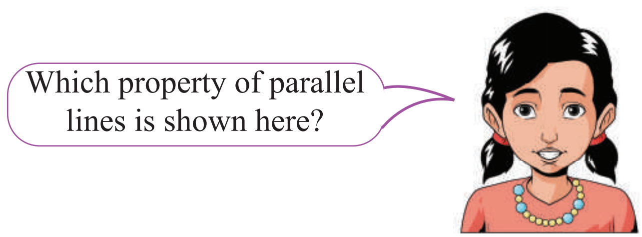
$\angle \mathbf{a} + \angle \mathbf{d} = \mathbf{180^0}$

$\angle \mathbf{b} + \angle \mathbf{c} = \mathbf{180^0}$



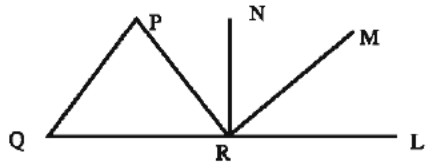
$\angle \mathbf{a} + \angle \mathbf{b} = \mathbf{180^0}$

$\angle \mathbf{b} + \angle \mathbf{d} = \mathbf{180^0}$



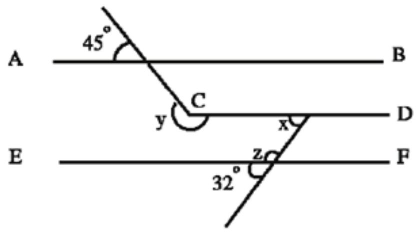
Solve the following problem using sticks or folding papers:

2.



In the figure, angle PQR=55^0, angle LRN = 90^0 and PQ is parallel to MR. Then what is the value of angle MRN?

3.



In the figure, AB, CD and EF are parallel to each other.

a) What is the value of angle z?

b) What is the value of angle x?

c) What is the value of angle y-z?

Properties of Triangle

In this part we will enclose an area with three sticks and discuss about its properties. You know from previous classes that an area enclosed with three sides is called a triangular area and the boundary of this area is called a triangle. Throughout the part we will consider three sticks as three sides and construct various triangles. After that we will find out some properties through various works. Finally, we will apply the properties to solve some problems.

Three sides of a triangle

We know, the line segments enclosing a triangular area are called sides of the triangle. The intersecting points of the sides are called vertices. By the directions of previous class, did you collect three sticks? Let's measure them and try to form a triangle.

At first fill up the following table by measuring the length of the sticks. Then try to construct a triangle using them.

Sl.	Stick 1	Stick 2	Stick 3	Can you construct a triangle? (yes/no)

Now fill up the relations of measures in the table and notice if triangle could be constructed.

Sl.	stick 1 + stick 2 >stick 3 (yes/no)	stick 2 + stick 3 > stick 1 (yes/ no)	stick 3 + stick 1 > stick 2 (yes/no)	Can you construct a triangle? (yes/no)

You can observe that, when we could construct a triangle, the sum of any two sides was greater than the third side.

Individual task: Explain in which cases we can form a triangle.

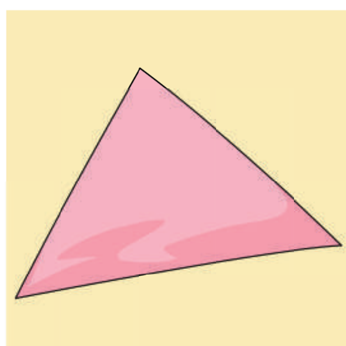
1. 1 c.m., 2 c.m. and 3 c.m.

2. 1 c.m., 2 c.m. and 4c.m.

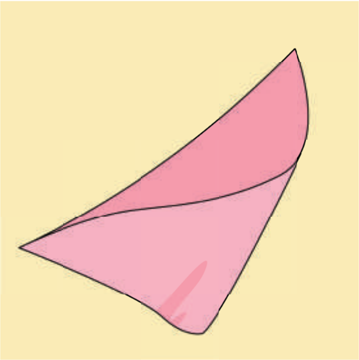
3. 4c.m., 5c.m. and 7c.m.

Median, angle bisector and perpendicular on opposite side

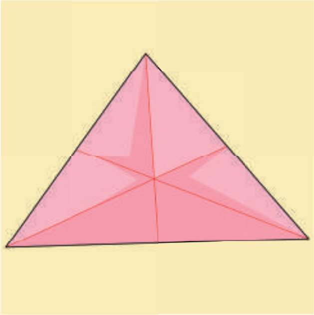
Construct a triangle cutting a paper like the following figure. Now we will discuss about various lines inside a triangle.



Median: The word 'median' is related with middle. So, we can say that this line is in the middle of something. Let's create a median at first following the steps given below:



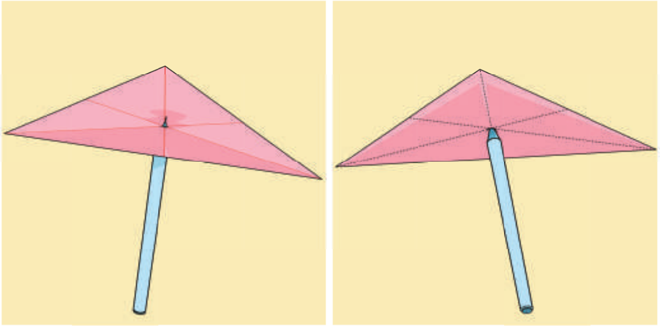
1. Like the given figure, match two vertices of a triangle. Notice the point in the middle where the crease is formed. Measure the distance of each vertex from this point. As the distances are equal, we can say that this is the midpoint of the side.



2. In the same way, find the median of all three sides and add them with the vertex opposite to the side by creating a crease. As we are adding the midpoints of the sides with opposite vertices, we call these lines as medians. In the following figure three medians of a triangle are drawn.

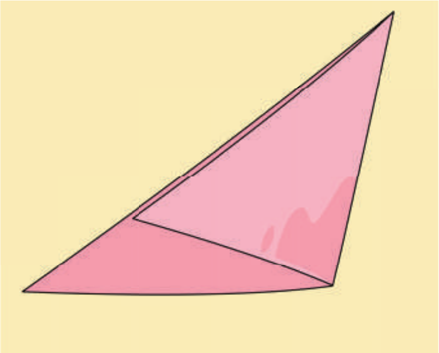
You can also draw median using a ruler. Find the midpoint by ruler and join with the opposite vertex.

Team work: Form some teams with 4-5 students in each team. Cut a triangle from an art paper or some thin board. Then find the midpoints of each side using a ruler and draw the medians. Notice that, there is an interesting property of the medians of a triangle. All three medians intersect at one point. Now tie a thread in that intersecting point under the supervision of your teacher. Now suspend the triangular shape using the thread, what do you see? The triangle is hanging parallel to the ground. Again, if you constructed the triangle using a hard paper, stick a pen or a pencil in the intersecting point. You will see that the triangle is remaining parallel.



As we can hold the triangle using only this point, we can say that the weight of the triangle is centered here. It means: this is the center of the mass of this triangle. For this reason, we call this point centroid.

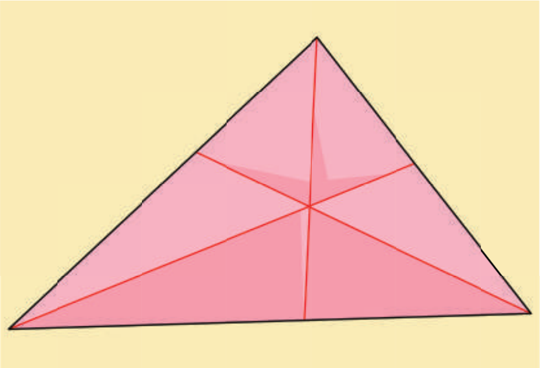
The medians of a triangle intersect at a specific point. This point is called centroid.



Angle bisector: From the name you can understand that these lines divide the angles of the triangle into two equal parts. Let's check how we can find the angle bisectors from a paper triangle.

1. Like the following figure fold in such a way that one side coincides with another. You will see that a crease is created which passes through the vertex. Now measure the two angles created at the vertex by the crease and check if there is any similarity between them.

2. You can see that the crease divides the internal angle at the vertex into two equal halves.

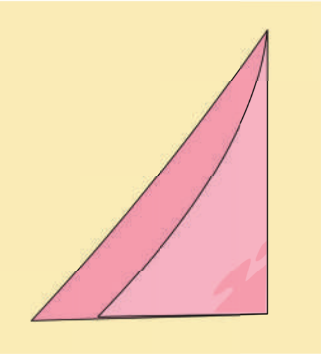


Individual task: Fold the triangle along other two vertices to find angle bisectors.

You can notice that, like the medians, the angle bisectors also have a separate point to intersect themselves; that means: the angle bisectors intersect at a specific point.

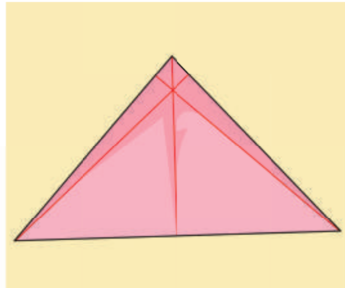
Perpendicular on the opposite side:

The perpendiculars drawn on the opposite sides from the vertices also represent some properties of a triangle. Let's follow the given steps to know how to draw them.



1. Fold the triangle in such a way like the following figure that the side on which we have to draw a perpendicular coincides with itself. The folded shape will look like a right-angled triangle from one side. What is the measure of the angle created at the crease?

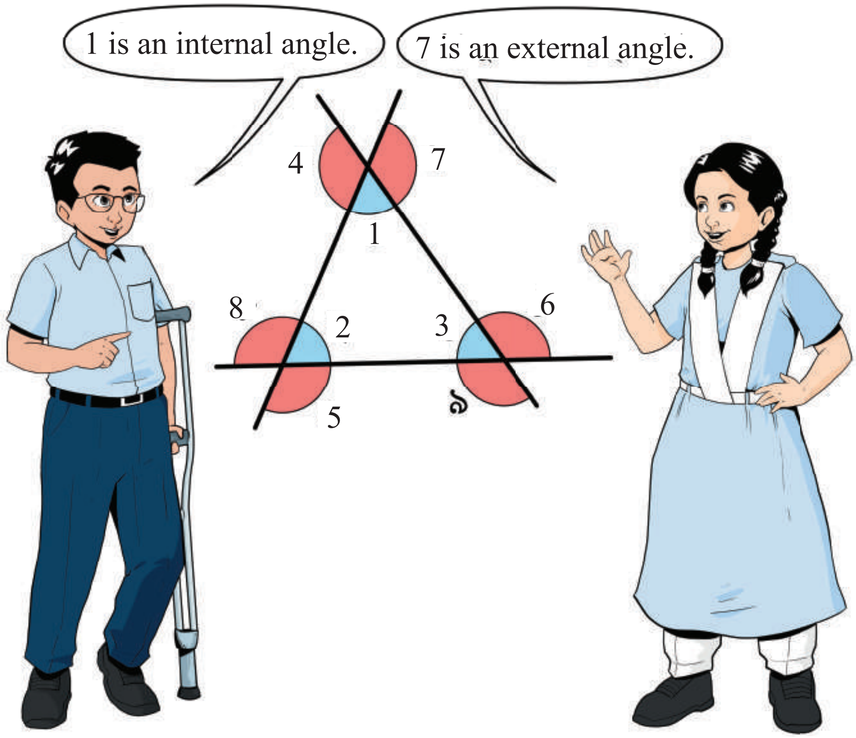
2. You can easily understand after measuring that we got the required perpendicular along the crease. In this way we will find all the perpendiculars and it will look like the following figure.



Task: Try to draw the perpendiculars on opposite side from each vertex in any other way.

Relation among the angles of a triangle

Now we will observe the angles of a triangle and check if there is any relation among them. Draw a triangle on your notebook such that the sides are extended like the following figure.

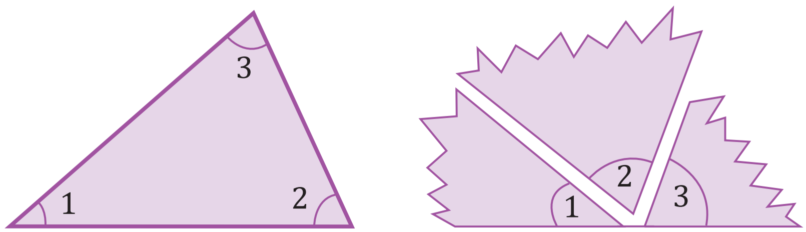


The angle marked as 1 is inside the triangle. We will say that this is an internal angle of the triangle. Similarly angle 2 and 3 are also internal angles of the triangle. Angle 4 and angle 7 are adjacent to angle 1 and they are vertically opposite. They are also formed with sides of the triangle, but they are outside of the triangular area. We will say that each of them is an external angle of the triangle. Similarly, angles 5, 8, 6 and 9 are external angles. If we are asked for the external angle associated with 1, we can measure 4 or 7. They are equal as they are vertically opposite to each other.

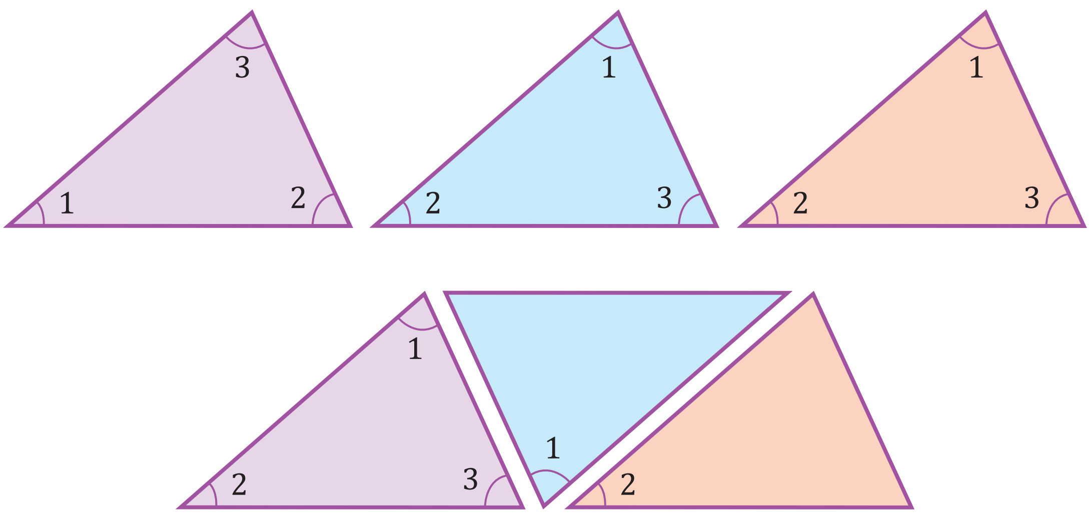
You already know about supplementary angles from your previous classes. If the sum of the measure of two angles is equal to two right angles, then they are called supplementary angles to each other. Notice that, the internal and external angles are adjacent and supplementary to each other. The internal angles not adjacent to a specific external angle are called opposite internal angles.

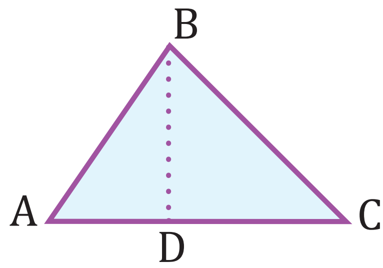
Let's search for some relations among the angles of a triangle now.

Individual task: Take a triangle made by paper. Break it into three pieces and arrange them like the figure. You can notice that a straight angle is formed.

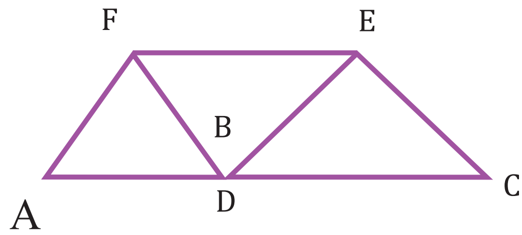


Individual task: Make three triangles of same size and shape and cut them in three different colors. Arrange them like the figure. You can also use white papers if color paper is unavailable. Is straight angle formed like the previous figure?

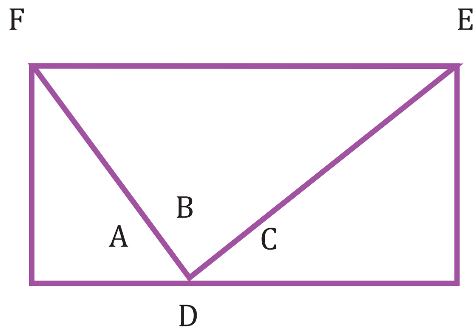




Individual task: Take a triangle ABC and construct a perpendicular on AC from B by folding AC. Let the perpendicular intersects AC at D.



Now fold the triangle like the figure in such a way that vertex B falls on the point D. Measure the line segments AE and BE. Do you observe any similarity/difference? Again check if there is any relation between AC and EF.



At last fold the triangle again by matching vertices A and C with D.

Team task: Make a team of five students and each of you draw a triangle on your notebook. Now fill up the following table using the measures of the angles of your triangles.

Sl	Internal angle 1	Internal angle 2	Internal angle 3	Sum of the internal angles	External angle adjacent to internal angle 1	Sum of internal angles 2 and 3	External angle adjacent to internal angle 2	Sum of internal angles 1 and 3	External angle adjacent to internal angle 3	Sum of internal angles 1 and 2

Do you see any special relation in the sum of the angles? Again, notice the relation among the external angles and their respective opposite internal angles. We can reach the following decisions from this information:

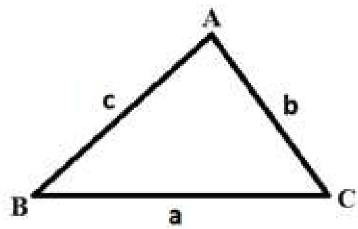
Sum of the angles of a triangle is equal to two right angles or 180^{o}.
Measure of any external angle is equal to the sum of two opposite internal angles.

Example: If all sides of a triangle are equal, we call it an equilateral triangle. All angles of an equilateral triangle are also equal. What is the measure of each angle?

Solution: We know, sum of the angles of a triangle is equal to two right angles or 180^0. As all are equal, measure of each angle will be $180^0 \div 3 = 60^0$.

Relation between sides and angles of a triangle

If we express the three vertices of a triangle by A, B and C, then we call it the triangle ABC. We express the angles by $\angle ABC$ (in short $\angle B$), $\angle BCA$ (in short $\angle C$), and $\angle CAB$ (in short $\angle A$). Generally the opposite sides of $\angle A$, $\angle B$ and $\angle C$ are expressed as *a, b* and *c* respectively. Notice the figure. The vertices and their opposite sides are marked there.



In the previous parts we observed some relations among the sides themselves and among the angles themselves. Now to check the relation among the sides and their opposite angles let's fill up the following table.

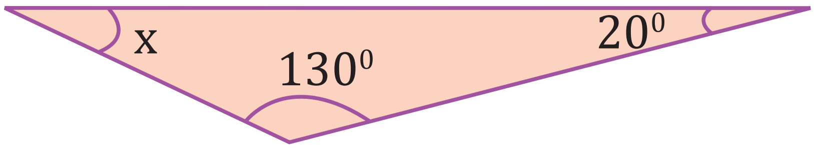
Sl.	Internal angle A	Internal angle B	Internal angle C	Side a	Side b	Side c	Greatest side	Smallest side	Greatest angle	Smallest angle

Did you notice anything special from the measures of the sides and angles? From the table you can know another important relation among the sides and angles of a triangle. It is:

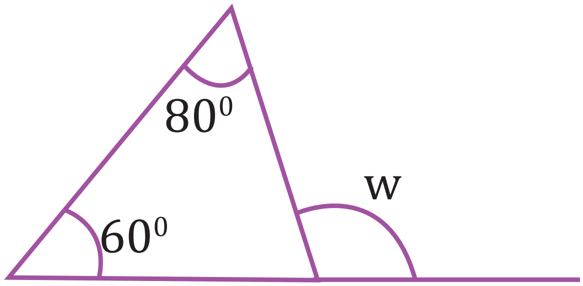
> The angle opposite to the greatest side is greatest and the angle opposite to the smallest side is smallest.

1. You are asked to draw a triangle whose sides are 4 cm, 5 cm and 10 cm. Can you draw the triangle? Explain in one sentence why you can or cannot.

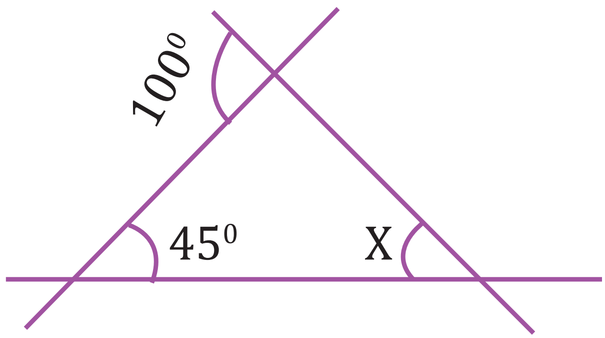
2. Find the value of angle x from the following figure (digits will be in English.)



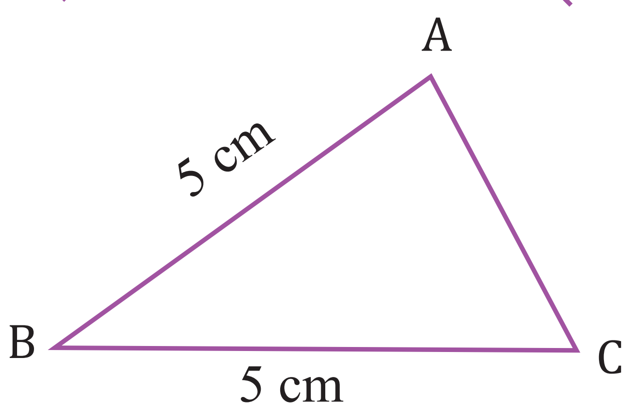
3. Find the value of angle w from the following figure (digits will be in English.)



4. What is the value of angle x in the following figure ? (digits will be in English.)



5. Joy has constructed a triangle whose sides are different in length than shown in the figure. Using this measurement, find out what is the greatest angle of this triangle. (digits will be in English.)



Congruence and Similarity

We see things of various sizes and shapes around us. Let's compare some of them.

Things	Size	Shapes	Weight	Comments
				
				
				
				

From the above table, you can see that some things are equal in all aspects. Again, some things are similar when we observe, but not equal. For example, if you compare two Mango leaves they look similar but their sizes may be different. On the other hand, two mathematics books are equal in all aspects. Same thing happens with two geometric shapes. Two geometric shapes may be equal in all aspects or there may be some difference in their sizes. The names for these concepts are congruence and similarity. We will explore these concepts in this part of the book.

Congruence

Among the things we compared just now, one comparison was of two mathematics books. Their sizes, shapes and weights all were equal. Some geometric shapes can also show this property. For example, if we compare between two triangles, we check if all three angles and three sides are equal or not. In this way if they are equal in all parts we say that they are congruent. The word congruent means meeting together. As the shapes are meeting together in all aspects, we are using this word.

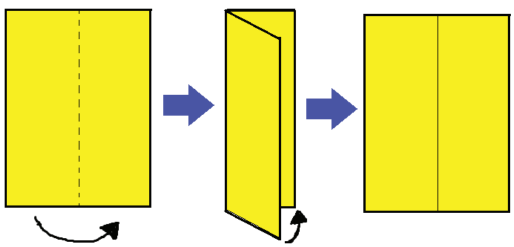
One way to check whether two geometric shapes are congruent is to measure all parts and check their measure. For example: measuring all angles and sides of a triangle. We can compare two geometric figures by putting one shape over another. Let's see a game to do that.

Paper Aeroplane

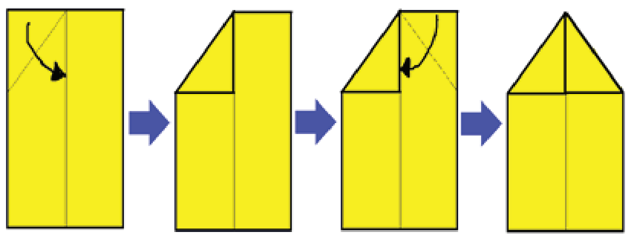
Now each of us will build and fly a paper aeroplane. After that we will find various geometric shapes in those aeroplanes.

Step 1: Each of you take a paper and put a mark on that so that you can later know your aeroplane. You will build your aeroplane using the marked paper.

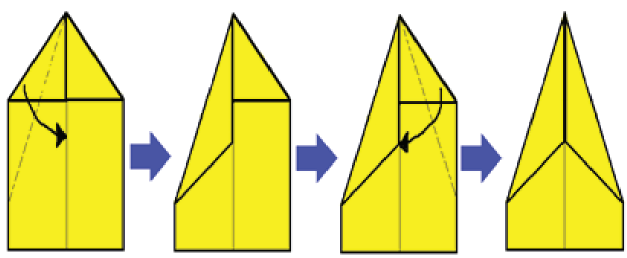
Step 2: At first fold the paper in the middle along the length. You will see a crease in the middle of the paper. If required check the process with your teacher more than once.



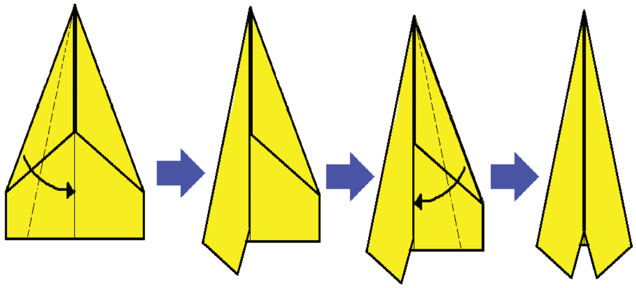
Step 3: Now fold the upper-left corner merging with the crease in the middle in the same way shown in the figure. In a similar way, fold the upper-right corner of the paper.



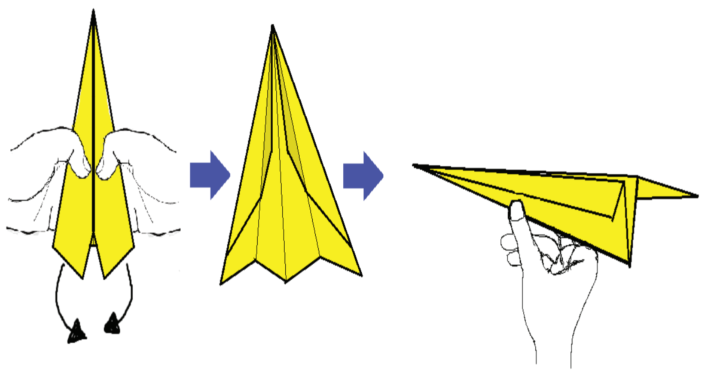
Step 4: Now again fold the left and right side of the paper in the same way as the figure.



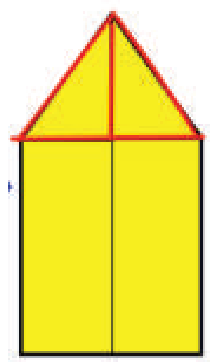
Step 5: Fold the shape both in left and right side for the third time merging it with the middle. Check from the figure how the folding is supposed to look like.



Step 6: Lift the whole paper to a small amount using both hands. Now fold it downwards along the middle line. Now hold the middle part using two fingers following the figure.



Step 7: Now we will have a competition of flying the planes. Following the teacher's direction, fix a place to throw the plane. You will come to the place one by one and throw it in the air. Mark a slightly distant place and try to throw the plane as close as possible to that place. The student whose plane will go closest to that place will be the winner.



Now let's observe the geometric shapes of different part of the plane. Check step 3 to find out that we found the shape below by folding the upper left and right corners. Find out various geometric shapes from the figure below. You can see that two triangles are marked for you. Measure their angles and sides and fill up the table below.

	1st side	2nd side	3rd side	1st angle	2nd angle	3rd angle
Left triangle						
Right triangle						

Task: Find out other geometric shapes from the figure. Create a table like below and fill up the table measuring the angles and sides of the geometric shapes.

Now find out various geometric shapes from the shape created in step 4. Measure the angles and sides and fill up the table below. As an example, two quadrilaterals are marked for you and a table is given.

	1st side	2nd side	3rd side	4th side	1st angle	2nd angle	3rd angle	4th angle
Left quadrilateral								
Right quadrilateral								

A sample table for other geometric shapes is given below:

Now take a new paper and proceed up to step 3 of building a plane. Cut the triangular shapes from both corners. Now fold it again like step 4 and cut the triangular shapes from both sides. Now for each pair of triangles, put one triangle over the other such that corresponding sides are one another. We are fully position one triangle on another, this is called superposition. We will say that one triangle is superposed on the other. In this way we can check any two geometric shapes for superposition. If they are superposed, we will say that they are congruent with each other.

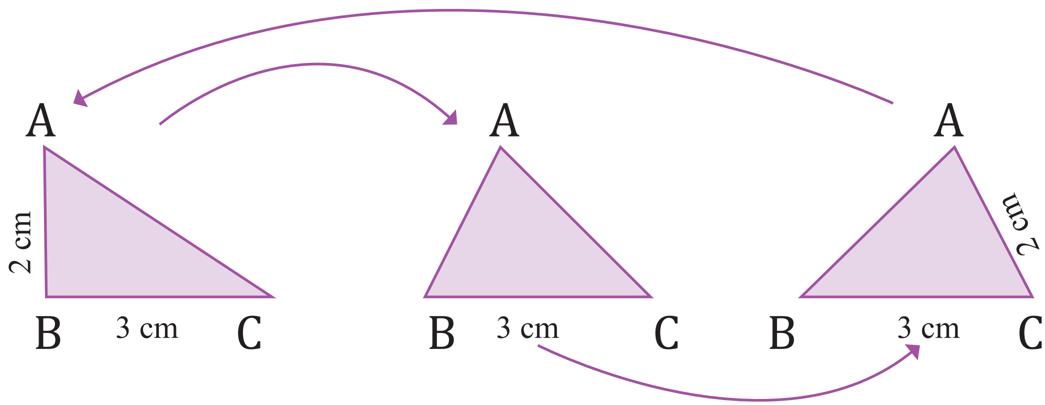
There is a benefit of working with triangles. We do not have to check all six parts. Again, we do not always have to cut papers and superpose to check for congruence. We can check some of the parts and decide if two triangles are congruent or not. Let's check what is the minimum information required to check for congruence of two triangles. We can also say it in another way. We will check for the minimum information required to get a specific triangle.

In your previous grade, you learned how to use protractor, ruler and compass. Now we will use them to construct some triangles using some given information and check if we get a specific triangle.

Task: Construct a triangle ABC using protractor and a ruler using given information:

1. The side BC is 3 cm.

2. The perpendicular drawn from A on BC is 2 cm.



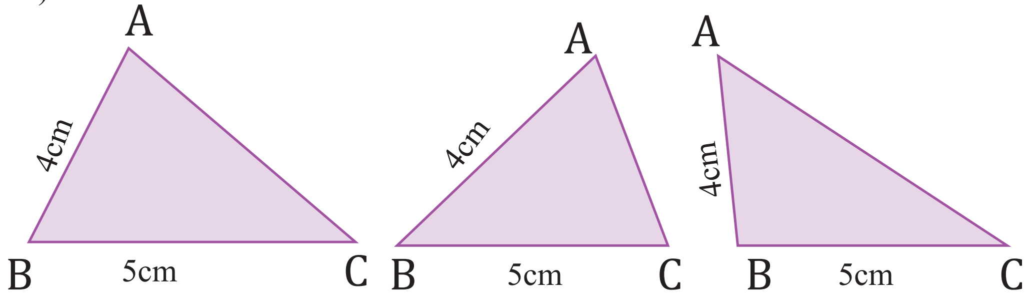
Notice that, the triangles are different due to different positioning of the perpendicular. The students in your class may get different triangles. That means, we can't draw specific triangle using only two information.

Now we try to draw triangle ABC using two other information.

1. The side AB is 4 cm.

2. The side BC is 5 cm.

(We are showing three possible triangles in the figure. It is possible to get more than that.)



You will notice that this time also different students are getting different triangles.

So, let's think the process from start. How can we construct a specific triangle using ruler, compass and protractor?

At first, we can draw a line equal to a given side BC.

Notice that we will get the specific triangle if we know the position of vertex A.

Now think which information is necessary to find the position of vertex A.

a. Which sides and angles will be used?

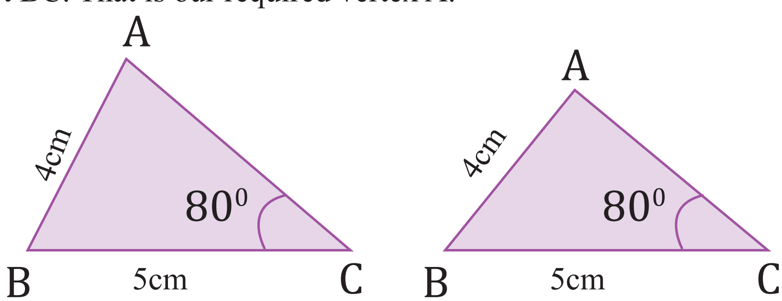
b. How many angles and sides will be used?

Notice that, in the above works we did not know the measure of any angle. In the first case we knew only a side and the perpendicular on it from the opposite vertex. In the second case we knew only the length of two sides.

So, let's try with three information about the triangle ABC.

1. The side AB is 4 cm.
2. The side BC is 5 cm.
3. The angle BCA is 40^0.

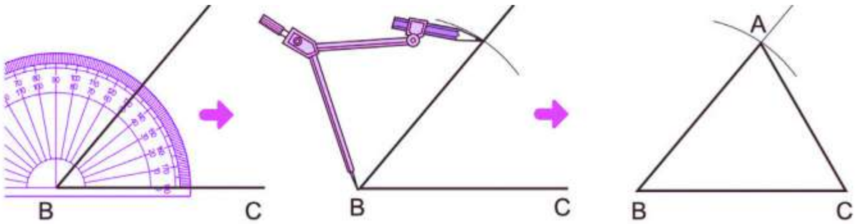
In this case first draw a 40^0 angle at point C of the given 5 cm. side BC. Then place the zero point of the ruler on B and check when the ruler matches the other side of the angle except BC. That is our required vertex A.



You can see that this time we didn't get a specific triangle too. We got two different triangles. So using these three pieces of information about the triangle also will not help us checking congruence of two triangles.

Now let's form three teams. Each team will try to construct one of the following triangles using given information.

Team 1: Take two sides BC=5 cm, AB = 4 cm and the angle between them ABC = 500. Everyone in the team draw the side BC = 5 cm. Then using a protractor, draw angle ABC = 500 at point B. Then cut off a 4 cm line segment from the side of the angle other than BC using a ruler or a compass. Name that point as A. So, we get AB = 4 cm



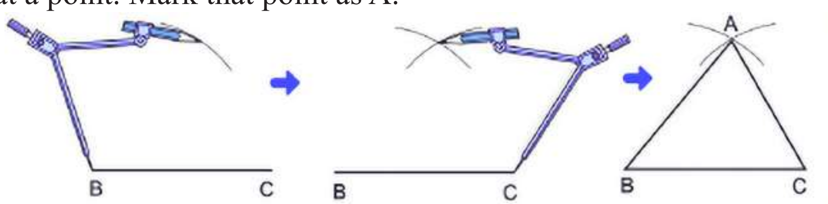
Join the points A and C. Now measure the side AC, angle BAC and angle BCA and write on your notebooks. Check the values with other members of the team. You will see that the values are the same for everyone. That is, the triangles are congruent for everyone.

Hence, we discovered an important information about congruence of triangles.

> If for any two triangles, any two sides and the angle between them are equal then the triangles are congruent.

That means we can construct a specific triangle if you know two sides and the angle between them.

Team 2: Take three sides BC = 7 cm, AB = 4 cm and CA = 6 cm. At first draw the line segment BC. Then use a compass to draw a circular segment at 4 cm distance from B. In a similar way draw a circular segment at 6 cm distance from C. The segments intersect at a point. Mark that point as A.

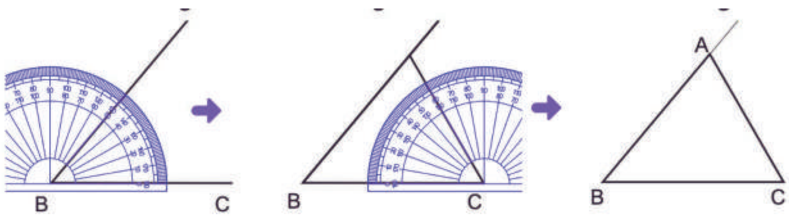


Now add the point A with B and A with C drawing straight lines. Three sides are drawn following a fixed measure. So, measure the angle ABC, angle BAC and angle BCA and write down in your notebooks. Then check it with other team members. You will see that the values are the same for everyone. That is, the triangles are congruent for everyone.

So we found the second condition of congruence of triangles.

If all three sides are equal for two triangles, then they are congruent.

Team 3: Take two angles $ABC=60^0$, $ACB=70^0$ and the side BC = 6 cm between them. At first draw the line segment BC. Then by the help of protractor draw angle $ABC=60^0$ at point B. After that draw angle $ACB=70^0$ at point C. Sides of these angles except BC will intersect at a point. Name that point as A.



Now measure the sides AC, AB and the angle BAC. Write the measurements in your notebook. Compare the values with others. You will see that the values are the same for everyone. That is, the triangles are congruent for everyone.

If two angles and the adjacent side of two triangles are equal then the triangles are congruent to each other.

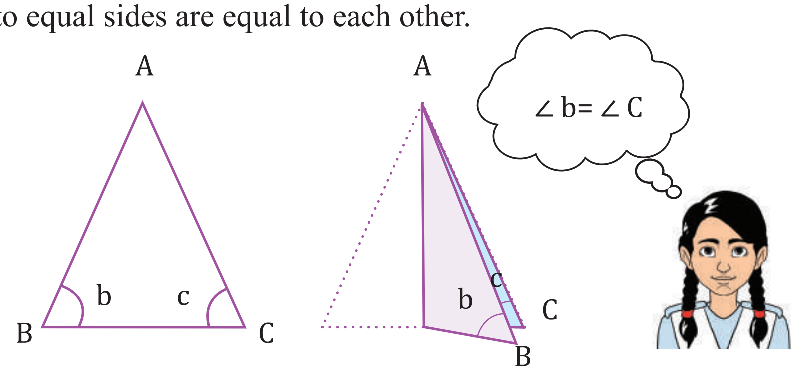
So we reached the third condition of congruence of two triangles.

That means we can construct a specific triangle if we know the value of two angles and a side.

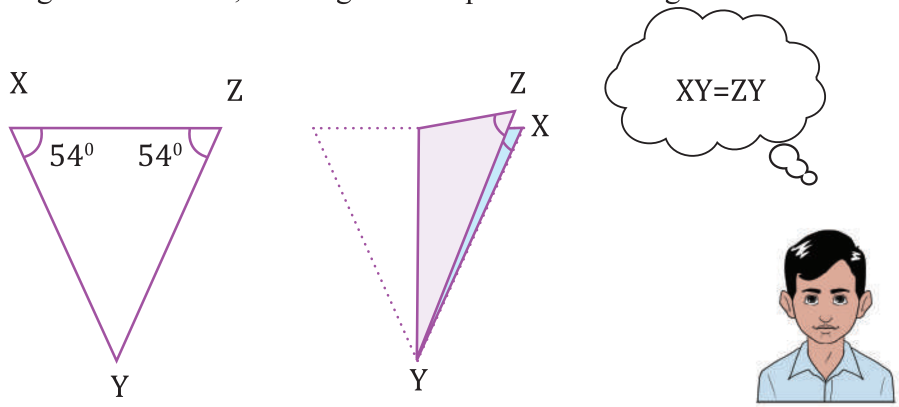
Let's think if we can find any other properties of a triangle logically using the conditions for congruence of two triangles. Using the relations of given sides and their angles, we will try to find some other relations. To do this we will try to find two congruent triangles.

1. Fold a isosceles triangle in the same way shown in the figure. What do you see?

Angles opposite to equal sides are equal to each other.

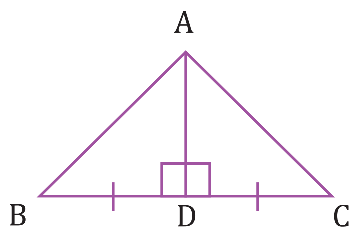


2. In the triangle XYZ below, two angles are equal. Is the triangle iscosceles?



Example: If two sides of a triangle are equal then the opposite angles of the sides are also equal.

Solution: We will try to divide the given triangle into two congruent triangles. Then it will be easy for us to show that the opposite angles are equal. Let us consider we have a triangle ABC like the figure below where AB = AC. We draw a median from A on BC which intersects BC at point D.



Now let's compare triangles ABD and ACD. We already know from the question that AB = AC. Again, since AD is a median, BD = DC. At last notice that the side line AD is common in both of the triangles.

All three sides of the two triangles ABD and ACD are equal and hence they are congruent.

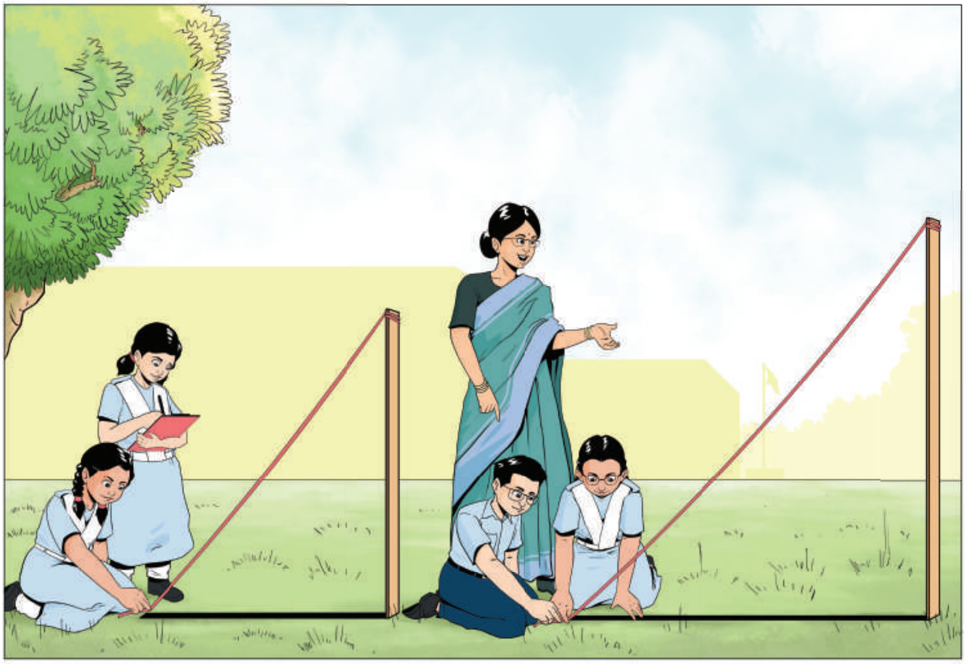
Thus the angle ABC and angle ACB are equal to each other.

So we see that if two sides of a triangle are equal then their opposite angles are equal. We call a triangle is isosceles triangle if two sides of it are equal.

Similarity

In the beginning we compared various types of things. There, some things had same shape but not the same size. These types of things are called similar. Let's explore through a team work when two geometric shapes are similar.

Team work: Take two sticks of different length, a scale, a long thread and a protractor. Now, whole class moves to a place where sunlight is available. At first measure the length of the sticks. Then, two of you hold the sticks vertically in the sunlight like the figure. You will observe that there are shadows of each stick. Now measure the length of the shadow. Finally hold the thread in a straight way from the upper endpoint of the stick to the last part of the shadow. Now measure the length of the thread for each stick.



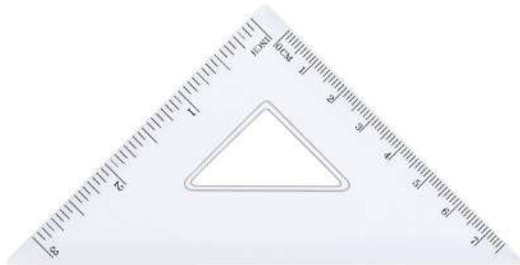
Now fill up the following table using the length of the stick, the shadow and the thread corresponding to each stick.

	Length of the stick	Length of the shadow	Length of the thread
1.			
2.			
Ratio of two lengths			

You can understand from the table that the ratio of the lengths is equal for both sticks and the shadows. So, if we know the height and the shadow of a smaller thing, we can find out the height of a bigger thing knowing its shadow. The geometric property that helps us doing this is called similarity. Here the triangles created by the sticks, their shadows and corresponding threads are similar.

Work: Measure the shadow of a stick of known length. Measure the shadow of the flagpole of your school at the same time. We now know that the ratio of shadow lengths is equal to the ratio of the stick lengths. Using this information calculate the length of the flagpole.

Let's bring out the following set square from our geometric boxes.



You can see two triangles in the set square, one is inside another is outside. Do they look the same? Though they look alike, they are of different sizes. Using the protractor from the geometry box, measure the angles of the triangles and fill up the following table:

Size of the triangle	1^{st} angle	2^{nd} angle	3^{rd} angle
Larger			
Smaller			

We can see from the table that the corresponding angles of the two triangles are equal. We can say that the reason for the triangles looking alike is that the angles are equal.

Task:

Fill up the following table measuring the sides of the triangles:

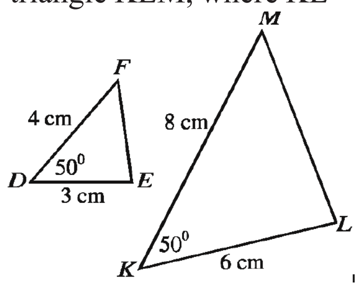
Length of a side of the larger triangle	Length of corresponding side of the smaller triangle	Side of larger triangle÷Side of smaller triangle	results

Write what we can decide from the results of the tables:

We can see that when two triangles are similar the corresponding angles are equal, and the ratio of corresponding sides is also the same. If the ratio is 1, then the similar geometric shapes become equal in all aspects. That means that they are congruent. So we can say that congruence is a special case of similarity.

Now let's check if we must know all angles and sides to check similarity or knowing only some angles and sides is enough. Let's get divided into three teams and perform the following tasks.

Team 1: Construct a triangle DEF, where DE = 3 cm, DF = 4 cm and between them angle EDF = 50^0. Construct another triangle KLM, where KL = 6 cm, KM = 8 cm and between them angle LKM = 50^0.

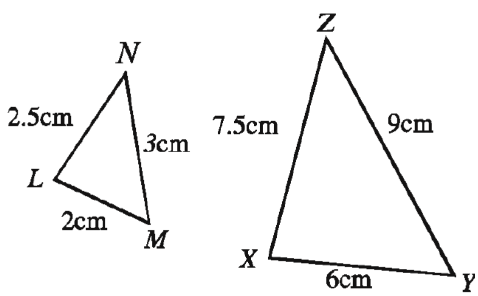


Measure the other angles and the ratio of the corresponding sides. Can we say that the triangles are similar?

So we get the first condition of similarity. It is:

If two sides of a triangle have the same ratio with corresponding two sides of another triangle and the angle between them are equal, then the triangles are similar.

Team 2: Construct a triangle LMN where LM = 2 cm, MN = 3 cm and LN = 2.5 cm. Construct another triangle XYZ where XY = 6 cm, YZ = 9 cm and XZ = 7.5 cm.

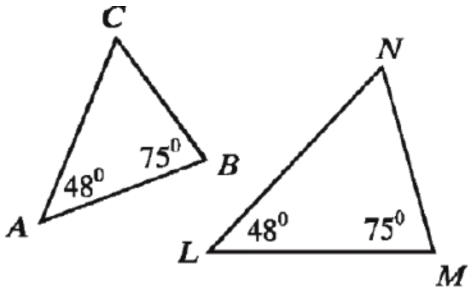


Measure the corresponding angles and the ratio of the corresponding sides. Can we say that the triangles are similar?

Hence we can state the second condition of similarity, that is:

> If all three sides of a triangle have the same ratio with corresponding sides of another triangle, then the triangles are similar.

Team 3: Construct a triangle ABC where angle BAC = 48^0, angle ABC = 75^0. Again construct a triangle LMN where angle MLN = 48^0, angle LMN = 75^0.



Measure the corresponding angles and the ratio of the corresponding sides. Can we say that the triangles are similar?

From the results of the 3^{rd} team we reach the 3^{rd} condition of similarity, which is:

> If two angles of a triangle are equal to two angles of another triangle, then the triangles are similar.

Game of four sticks

Now we will construct various types of quadrilaterals using four sticks.

Step – 1: Put marking on 4 sticks keeping a gap of 1 cm between each mark with a scale. You can also use a pipe of drinking juices. (figure 1)

Join one end of two sticks using a thread like the figure. (figure 2)

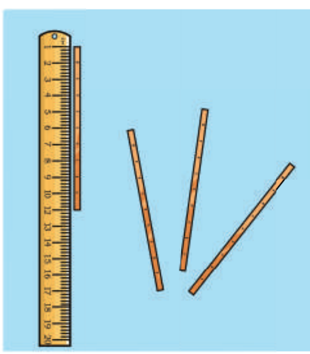


Figure 1

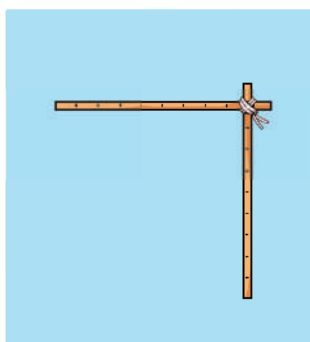


Figure 2

Step – 2: Now, join another stick with any of the two sticks using a thread. (Figure 3)

Finally join the last stick with the remaining two open ends in the same way. (Figure 4)

Step – 3: We will play games of creating various geometric shapes using this apparatus. The games are like this:

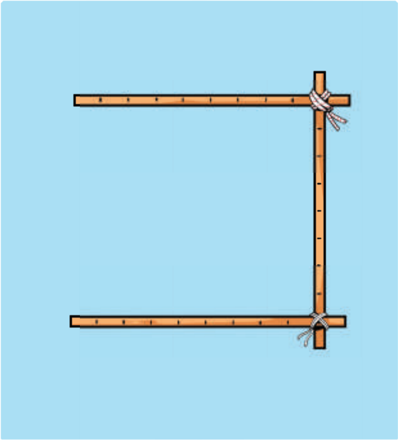


Figure-3

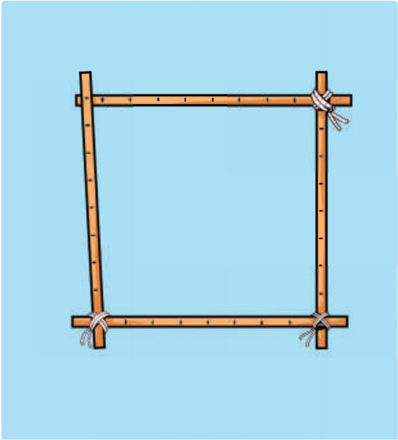


Figure-4

1. Keep each of the sticks three marks away from adjacent ones. Keep one of them perpendicular with nearby one and hold it up. All sides are 3 cm and the sticks are perpendicular to each other. So the geometric shape here is a square whose sides are 3 cm. Put the geometric shape on a paper and draw it.

2. Now from this square keep any two sticks at an angle of 60 degrees and hold it up. So this will be a rhombus whose sides are 3 cm and one of the angles is 60 degrees. Put the geometric shape on a paper and draw it.

3. In a similar way make a rectangle whose two sides are 3 cm and 4 cm and hold it up. Put the geometric shape on a paper and draw it.

4. Following the same method make a parallelogram whose two sides are 3 cm and 4 cm and an angle is 60 degrees. Hold it up to show your teacher and Put the geometric shape on a paper to draw it.

(Going to bubble: We created some special quadrilaterals till now. Let's try to construct some quadrilaterals whose sides and angles are given –

1. Construct a quadrilateral whose four sides are 3, 4, 5 and 6 cm. Hold it up to show to the whole class. Do the quadrilaterals look same for the whole class? Put the geometric shape on a paper and draw it.

2. Construct a quadrilateral WXYZ where WX = 5 cm, XY = 4 cm, YZ = 3 cm, ZW = 5 cm. Notice that, in the first game there was no name of the vertices. But here we named the vertices so that the sides go to a definite orientation. Hold it up to show to the whole class. Do the quadrilaterals look same for the whole class? Put the geometric shape on a paper and draw it.

3. Construct a quadrilateral KLMN where angle K = 45^0, KL = 3 cm, LM = 4 cm, MN = 2 cm, NK = 5 cm. Hold it up and check if the quadrilaterals look same for the whole class.

Notice that, everyone got the same quadrilateral only in the third construction. So what can we decide from these three games in step 4?

If four sides and an angle for a triangle are given, we can construct a specific quadrilateral.

Remember, two triangles are similar if corresponding angles and the ratio of corresponding sides were equal. The same is true for quadrilaterals. Corresponding angles should be equal and corresponding sides should have the same ratio between two similar quadrilaterals.

However, we do not have to check all four sides and all four angles to check similarity between two quadrilaterals. We could construct a specific quadrilateral when we had four sides and an angle. So two quadrilaterals may be similar if four sides have the same ratio and only one corresponding angle is equal. Let's check the assumption by constructing two such quadrilaterals.

Team work: Form a team of 3-4 members and do the following work.

Step 1: With the help of four stick apparatus, construct a quadrilateral ABCD where angle A = 50^0, AB = 3 cm, BC = 3.5 cm, CD = 2 cm, AD = 2.5 cm. Draw the figure on your notebook.

Step 2: In a similar way, construct a quadrilateral EFGH where angle E = 50^0, EF = 6 cm, FG = 7 cm, GH = 4 cm, EH = 5 cm. Draw the figure on your notebook.

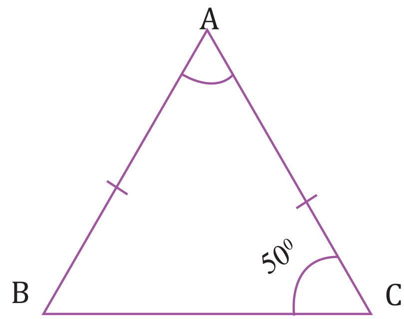
Step 3: Measure the rest of the angles. Are corresponding angles equal?

Step 4: Do the figures of the quadrilaterals look similar?

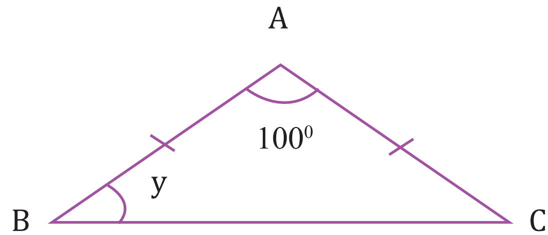
We can conclude that if the ratio of corresponding sides is the same and any pair of corresponding angles are equal then two quadrilaterals are similar.

Exercises:

1. In the figure, ABC is an isosceles triangle where AB=AC. What is the measure of the angle marked as w?



2. In the figure, ABC is an isosceles triangle where AB=AC. What is the measure of the angle marked as y?

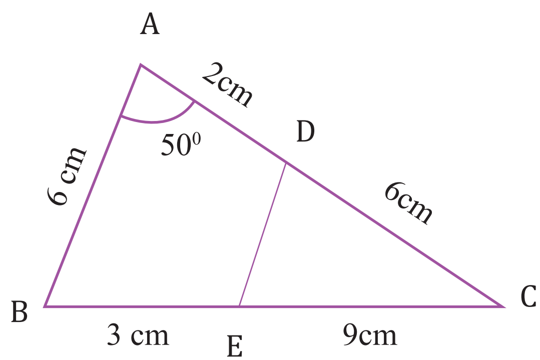


3. In the figure AB and DE are parallel to each other. Answer the following questions using this information.

(a) What is the value of angle ADE?

(b) There are two similar triangles in the figure. Find out who they are. Why are they similar?

(c) Using the properties of similar triangles, find out the length of DE.



The Story of Binary Numbers

Guessing game

Let's play a guessing game. The game is to guess the name of favourite book or famous person or movie. Let me tell you the rules. One will go to the front of the class by lottery and will remember the name of a favourite book, a famous person or a movie. All other classmates will ask questions and try to get the right answer from him/her. But there are some conditions. S(he) can't utter anything or use any sign language to give the answer to any questions. S(he) will have a torch or switch of a light and must answer by turning on that light. S(he) will turn the light on once if the answer to the question is yes. If not, he will not turn the light on.

Suppose, Salma thought of the name of national poet Kazi Nazrul Islam. Now see how everyone is asking questions to Salma.

-Is this a book? Salma will not turn on the light. - Is this a person? Salma will turn on the light. - Is he a writer? Salma will turn on the light. - Is he still alive? Salma will not turn on the light. - Did he write poetry? Salma will turn on the light.	- Was he born in Barisal? Salma will not turn on the light. - Was he born in present day West Bengal? Salma will turn on the light. - Is she a woman? Salma will not turn on the light. - Is he the poet Rabindranath Tagore? Salma will not turn on the light. - Did he die in Dhaka? Salma will turn on the light. - Is he the rebel poet Kazi Nazrul Islam? Salma will turn on the light.

How was the game?

Now think about other ways to say yes or no besides turning the light on fill up the table below:

With head or hand gestures
Using a paper with 'yes' on one side and 'no' on the other side.

Well, have you noticed that you used a signal during the game? If you needed to say yes, you turned on the light and if you needed to say no, you kept the light turned off. You have made a very difficult decision by simply using yes and no with such signals. A little more attention tells you that light is on when you switch

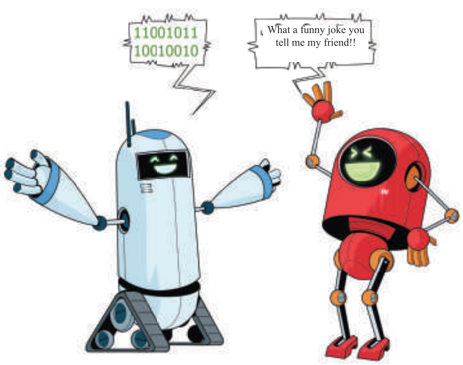
on to confirm the presence of electricity. So presence of electricity means yes and absence means no. Now mathematically, if we assume 1 for yes and 0 for no, then presence of electricity means 1 and absence means 0. That means you could come up with the correct answer using only 1 or 0 and reach a conclusion. It might have taken a lot of time, but there was no need to know or understand anything else except yes or no.

So there are two signals □ and □in the method of calculation of machines.

Now let me tell you something funny. The things you have seen around you like computers, televisions, mobile phones, calculators, all of them operate just like the guessing game. That means all works are done, using only yes and no; that is, 1 or 0. Surprising, isn't it? The fact is that we play video games on the computer, watch movies and write using the signals of the presence or absence of electricity in these devices. However this signal is not limited to a single yes or a single no. A large signal is created with the combination of a many yes or no that is, 1 or 0. But there is no other signal except these two.

Now, fill in the following table with the signals or digits we generally use to count or write a number.

For example, we use the decimal number system constructed with ten digits from 0 to 9 for doing math or calculations. This is not the case with computers or electronic devices. But what to do; they have to do all the works with 0 and 1.



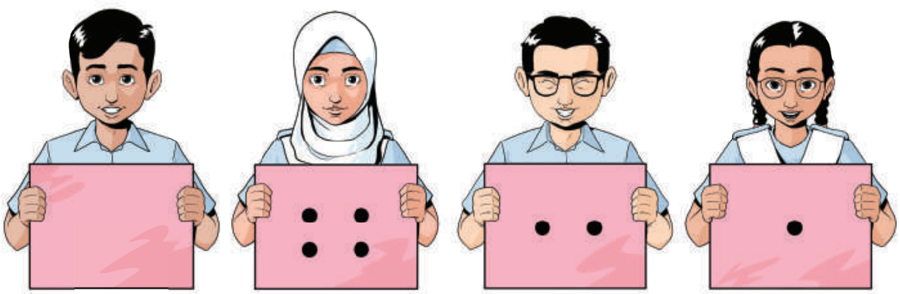
We call the symbols 0-9 as digits in the decimal number system. So, in Binary System, 0 and 1 are called Binary Digit. It is abbreviated to Bit taking Bi from Binary and t from Digit without saying Binary Digit again and again. We also write বিট in Bangla.

The binary number system has no other digit without 0 and 1. But there are many other number systems. Have you heard of the Piraha tribe in Brazil? They live deep in the Amazon forest. They have no connection with civilization at all. Their alphabet, vocabulary and method of counting are also very limited, science is out of question. They can't count more than 1 and 2. Any number greater than 2 is called 'many' by them. Interesting, isn't it?

Now, what if we can learn to count in binary system? Let me tell you one thing in advance, a good understanding of binary system will also help you understand how computer works. Not only that, you can solve many problems of computer by yourself. So let's get familiar with binary number system.

Count the dots in card

We shall understand how computer calculates through the following game.
At the beginning of the game, any four of you go to the front of the class and face others. Each of you will have one big card in your hands. Now draw a dot on the first person's card. In this way, draw two dots on the second person's card and four dots on the third person's card.



BrainStorming

Now think and tell how many dots will be there in the card of fourth person and how will you determine the answer?

....

....

....

If you understand this sequence, you will be able to tell the number of dots of the card of 5th, 6th, 7th…or any of your friend in that way. Now fill in the blank of the following:

Relation between the number of dots in each card with the previous card ______

Do you remember the rule of turning on the torch in the 'guessing game' at the beginning? 1 for the light on and 0 for light off. Just like that there is a rule also in this new game.

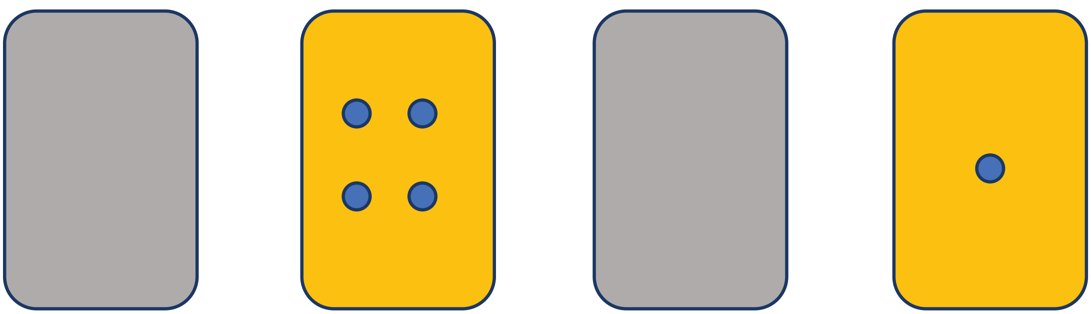
Rule of Game:

a) We will call the cards 'on card' for which the dot is visible. We can mark an 'on card' with ☐

b) We will call the cards 'off card' for which the dot is not visible. We can mark an 'off card' with ☐

Now, we know all the rules. Let's start counting.

Putting the 'on cards' sequentially and counting the number of dots on them will be our result. Look at the following picture- Total two cards are on and the rest are off. There is one dot on the first card, second card is off, 4 dots are on the third card and the fourth is off. The same card can't be used more than once. That means suppose you want to use the card with two dots twice. This is not possible. You have only one two dots card.



4th 3rd 2nd 1st

Look at the previous picture. Now write 'on' or 'off' below the cards and fill in the following blanks with 1 or 0 accordingly.

Order of card	4^{th}	3^{rd}	2^{nd}	1st
On or Off				
1 or 0				

Binary number

Decimal number

That means, Binary expression of decimal number_____ is 0101.

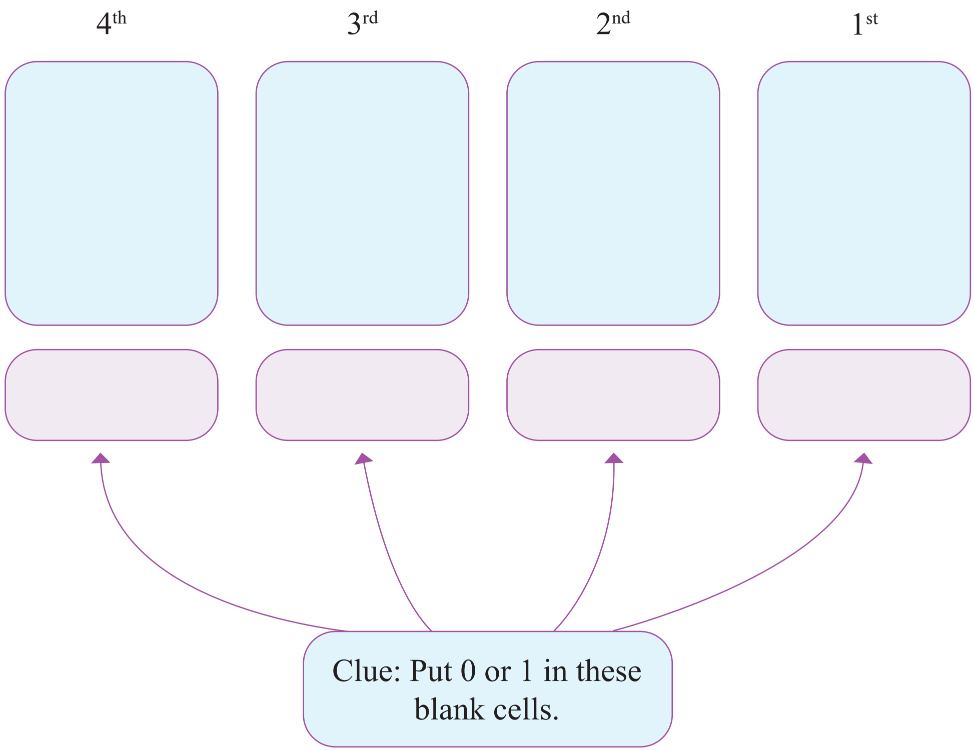
Now think another way.

- Observe carefully, we have no such card that has five dots.
- We have to use more than one card to make 5 dots.
- There are 8 dots in a card for the number greater than 5.
- But, we can't make 5 with 8 dots.
- In that case, 3^{rd} card has 4 dots which is less than 8 and we take it.
- Now think, how many dots are needed with 4 to make 5?
- Just 1, isn't it? We have a card with one dot – the 1^{st} card.
- So, we can make 5 with the 3^{rd} and 1^{st} card remaining 'on'.
- There is no problem with the rest of the cards remaining 'off'. That was done.

So, step by step, we found out by which cards would be on or off to get 5 dots. This method to solve a problem step by step is called ALGORITHM.

Pair task:

Now find out how decimal number 3 is expressed in binary system using card and dot. You can use the following table. Cards are left blanks to facilitate placement of dots. Place the right number of dots on the right cards and fill in the blanks below the cards:



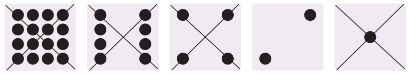
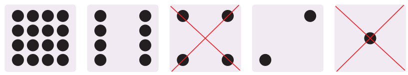
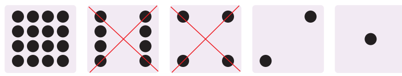
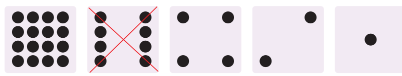
We have gone through the basic steps of counting binary numbers by counting the dots on the cards.

You have already learnt about binary digit or bit. A card represents one bit in the game of card. Since we have used total four cards, so first card is first bit, second card is second bit. Thus four cards represent 4 bits in total.

For example, 2435 is a 4 digit decimal number. Similarly, 4 digit number in binary system can be expressed using the states of four cards (on or off that means 1 or 0). For example, 5 is a 1 digit number in decimal number system. And binary expression of 5 is 0101. This is a 4 digit binary number or has 4 binary digits . SO it's called 4 Bit number.

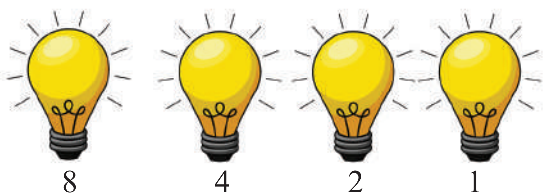
Individual task:

Fill up the blank cells of the following table using correct decimal number, card or binary number.

Number		Binary number
2		00010
5		
3		
		01100
19		
8		01000

Counting binary number without using card

In using cards, you have seen that when dots are visible the card is considered as 1 and if not visible the card is considered as 0. And the number of dot on each card is double the number of dots on previous card. If so, then why don't we assume only on or off? And what's better than a light bulb in explaining on-off. So let's see whether the same counting can be done without dots. In the picture below Bulbs are kept on instead of card and number is used directly instead of the number of dots. All of the places from 1st to 4th are 'on' in the above picture. Now think about it and circle the correct answer of the following questions.



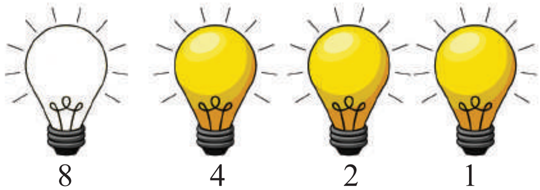
Quiz

1) Which number is expressed in binary in the above picture?
a. 1011 b. 1111 c. 1101 d. 1000

2) What is the decimal value of the binary number shown in the figure above?
a. 11 b. 10 c. 15 d. 16

Solve the following problems to make this more clear:

Problem 1:From the picture below Determine binary and decimal number and write in the blanks.



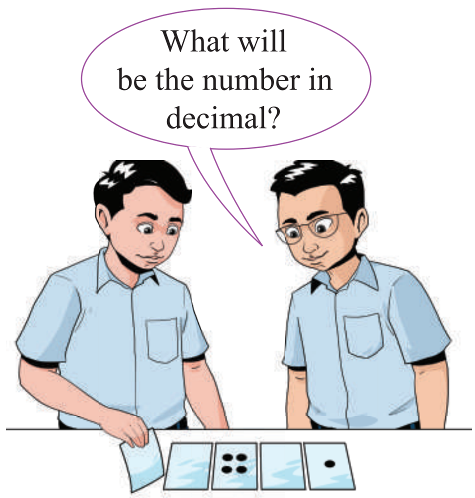
Binary:	Decimal:

Problem 2: What will be in decimal form of the binary number 1101?

Problem 3: What will be in binary form of the decimal number 13?

Problem 4: What is the number of bits of 101 in binary??

Problem 5: What will be in binary form of the decimal number 12? What is the number of bits of that number?



Brainstorming

Brainstorm and give the answer to the following questions quickly.

1. What is the highest number you can count using 4 bit in binary? What is that number in decimal form? [Write answer in the following blank space. Four bulbs have been drawn for your convenience to count. You can identify them] 
2. What is the maximum number you can make using 2 bits in binary? What is that number in decimal form?
3. What will be the number of bit in binary form of the decimal number 4?
4.What is the maximum number you can make using 5 bit in binary? What is that number in decimal form?
5. How many dots are there in the eighth bit?

Team task:

Making groups of four, you determine binary values of the numbers from 0 to 15 using card and bulb.

Let’s think more about it:

If you can determine the number of dots of extreme left card for different number of bits and largest possible number using those bits, then it will be easier for you to solve the problems of the previous page.

You can write the answers easily in the following table. Some are filled in for you.

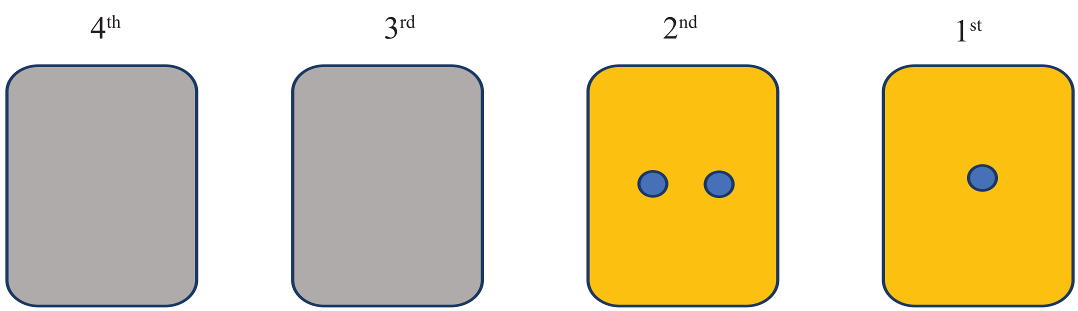
Number of bits (Number of Card)	Number of dots of extreme left card	Highest possible decimal number to create
1	1	1
2	2	3
3	4	7
4	8	
5		
6		
7		
8		

You have learnt to determine the highest number of each bit in the previous exercise.

Quiz

Observe the above table carefully. Now tell, is there any relationship between number of bits and the possible largest decimal number that can be written with those bits? Can you create any formula to find largest possible decimal number easily from the number of bits?

However it is possible to get numbers in each bit separately except the highest. It needs a little understanding. It is easier to understand using card. Look at the following picture.



The question is: What numbers can be made using up to the second bit?

Have you included 0 in the numbers of those you can make?

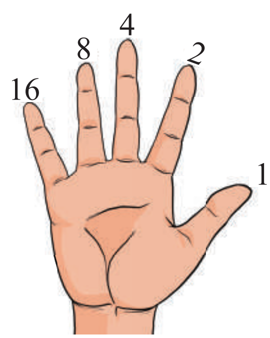
Then how many numbers are made including 0?

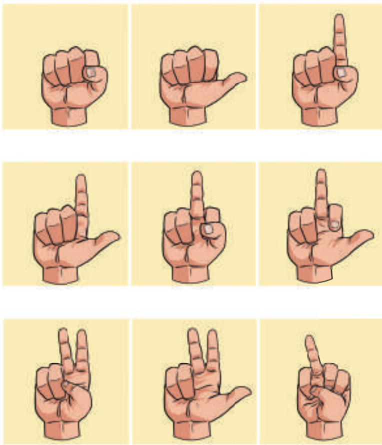
Now, fill in the following table.

Number of bit (Number of Cards)	How many total numbers are possible to get?(Including 0)
1	2
2	4
3	
4	
5	
6	
7	
8	

Binary counting using finger of hand

See, how much effort was necessary for us to learn counting a new number system. We used cards, bulbs and learned on-off. But when we calculate in decimal system, we feel free to count using fingers. Thinking simply, we can count up to 10 in decimal system using fingers of two hands. So, won't it be easy if you can count binary numbers using your fingers of hands? It would be great to use your fingers too in binary calculations when you don't have a notebook, pen or card handy, right? Do you remember the concept of on-off? In the picture below, finger raised or up means on. And if you keep it down, it is off.





First we use the fingers of the right hand. Think of your thumb as the 1st bit. Let the index finger be the 2nd bit. The middle is the 3rd bit. Ring finger is the 4th bit. And the little finger is the 5th bit. You also know how many dots are on which bit. Earlier you solved yourself the maximum binary number that can be calculated with 5 bits

Now it's your turn

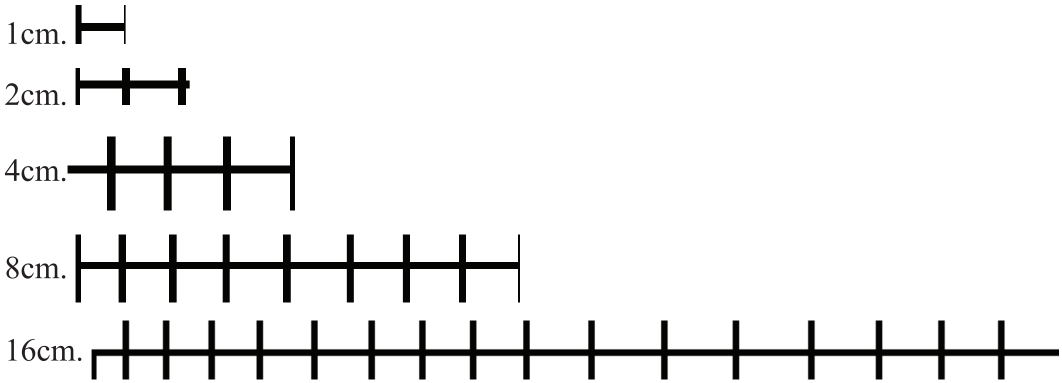
Count from 0 to 31 as shown above. Repeat this process until you find it easy. Share the method with your friends after doing it yourself.

Hints: Here the english word "up" means finger raised.

0	0					
1	1					up
2	2				up	
3	2+1				up	up
4	4			up		
5	4+1			up		up
6	4+2			up	up	
7	4+2+1			up	up	up
8	8		up			
9	8+1		up			up
10	8+2		up		up	
11	8+2+1		up		up	up
12	8+4		up	up		
13	8+4+1		up	up		up
14	8+4+2		up	up	up	
15	8+4+2+1		up	up	up	up
16	16	up				
17	16+1	up				up
	etc...					

Individual Task:

1. The challenge of measuring length:



The length of 1 cm, 2 cm, 4 cm, 8 cm, and 16 cm. are shown in the above figure. Cut papers / sticks equal to these lengths. Then taking them just once, see if you can measure each length from 0 cm. to 31 cm. Write in the table below how to measure.

Length(cm.)	**16 cm.**	**8** cm.	**4**cm.	**2**cm.	**1**cm.
0					
1					
2					
3	No	No	No	Yes	Yes
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					

16					
17					
18					
19					
20					
21					
22					
23					
24					
25					
26					
27					
28					
29					
30	Yes	Yes	Yes	Yes	No
31					

Mina has got the following ideas while making this table. Fill in the table with reasons whether you agree with Mina's ideas. (One is done for you.)

Idea of Mina	Do you agree with Mina?	Reasons
It is not possible to measure the length of 25 cm.	No	16 cm.+ 8 cm.+ 1 cm.= 25 cm. So it is possible to measure the length of 25 cm.
A length of 2 cm. is not necessary to measure a length of 12 cm.		
A length of 8 cm. is not necessary to measure a length of 22 cm.		
A length of 16 cm. is not necessary to measure a length of 15 cm.		
It is possible to measure up to 12 cm using the length of 1cm., 2 cm. and 4 cm.		

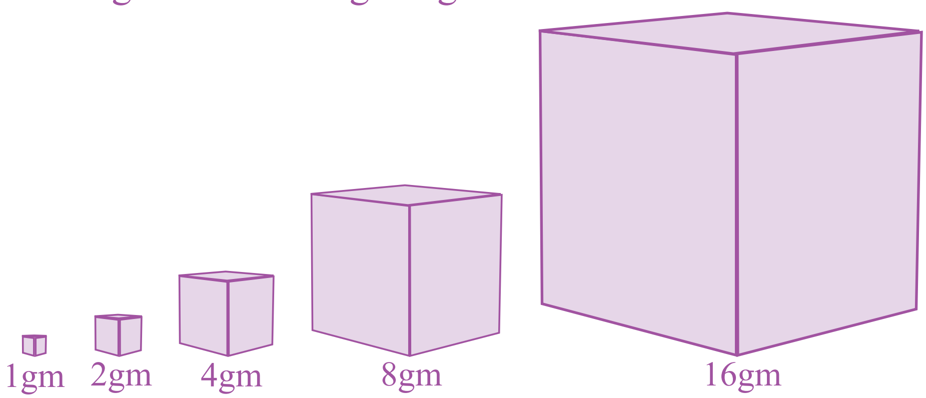
Observe, 16 cm+ 8 cm+ 1 cm= 25 cm. Again binary expression of 25 is 11001. Can you find any similarity of binary number with **the challenge of measuring length?** Look at the table of making length from 0 to 31 cm. again. Can you make any length more easily using binary number now?

If you can, then fill in the following table in that way.

Length (cm.)	Binary expression	16 cm.	8cm	4 cm.	2 cm.	1 cm.
25	11001	Yes	Yes	No	No	Yes
		1	1	0	0	1
11						
22						
23						

So, you get that although computer language is binary, it is not limited only there. Rather many problems can be solved easily using binary. You only have to observe and find where the concept of binary can be applied.

2) The Challenge of measuring weight:



1 gm, 2 gm, 4 gm, 8 gm and 16 gm are shown in the above figure. Then taking these weights just once,see if you can measure each weight from 0 gm to 31 gm. Show making a list how to measure like in 'the challenge of measuring length'. In this case, have you found any easy method?

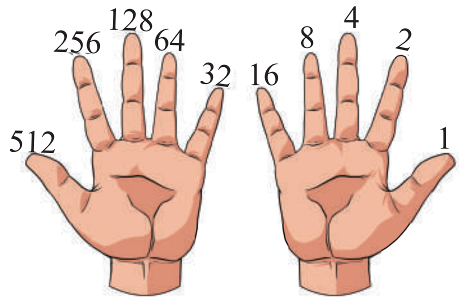
Your answer:

(Clue: You can look at **'the challenge of measuring length'**)

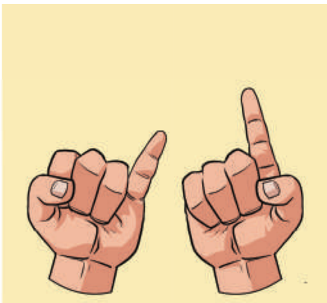
3) Binary Toy/Machine

We could count from 0 to 31 using 5 fingers of one hand in the part 'Binary counting using finger of hand'. However if you want to work with a very large number?

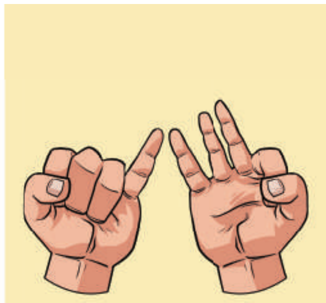
Use left hand:



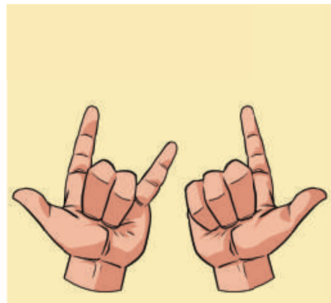
Now we can count in the following way using 10 fingers.



32+2 = 34

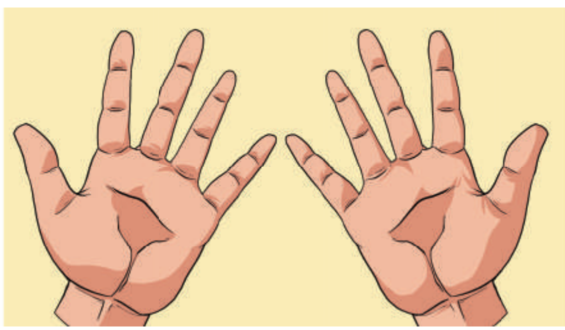


32+16+8+4 = 60



512+256+32+2+1 = 803

If we take all the 10 fingers?



512+256+128+64+32+16+8+4+2+1 = 1023

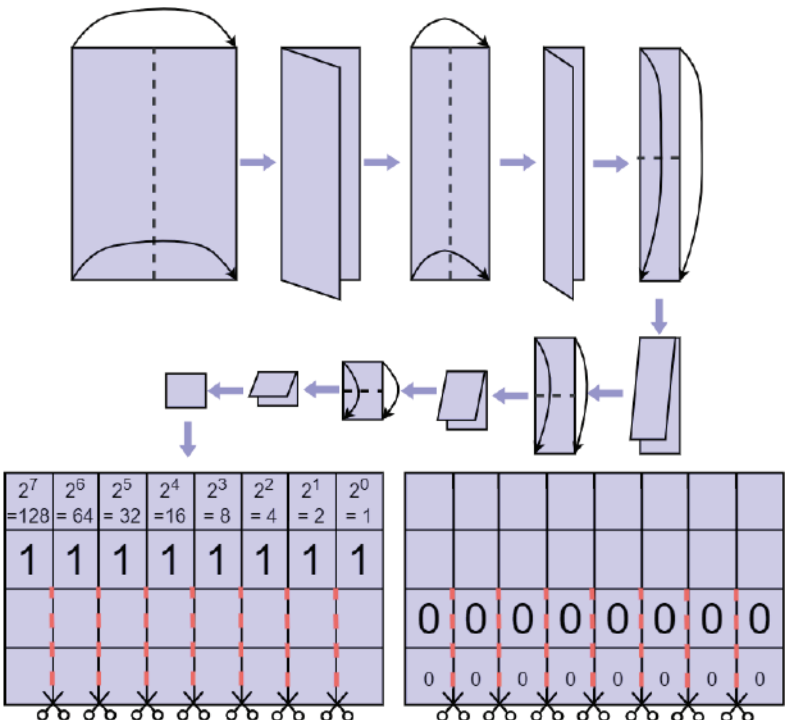
However, let you have to count up to 2022. Then it is not possible using two hands also. What do you think to do in this case? Write it.

1. We can count toes as well as finger.

2. We can call a friend also.
3.
4.
5.

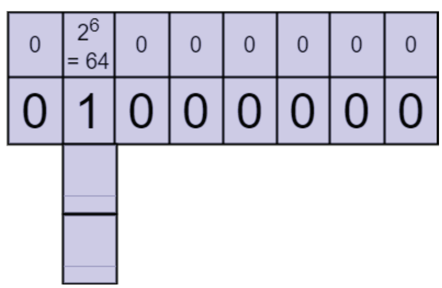
But you can make a nice toy/machine using paper yourself in your home by which one can express or transform the decimal number to binary number.

In the following pictures step by step direction is given about **How to make the toy/ machine**. Let's make the toy/machine following the steps. Teacher will help you if necessary.



front side of paper   back side of paper

The machine has been made. Now you need to know how to use it. Do you remember that we transformed 64 from decimal to binary using fingers? The following picture describes how easily 64 can be transformed from decimal to binary using this machine step by step.



3) Birthday magic trick

Majedur is a magician. He can tell anyone's birthday in the blink of an eye. He has five cards. If someone identifies the card that has his/her birthday, Majedur can instantly tell his birthday like a magician. But how does he do that?

Card 4			
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

Card 3			
8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

Card 2			
4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

Card 1			
2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

Card 0			
1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

Binary Candle or general candle on cake

We generally use one candle for each year on the cake of birthday.

Every candle will either be lit or extinguished. We can express your age in binary system using this. For example, binary form of age 14 is 1110. If you want, you can express it using candle.

Encourage people to use binary candles on their birthdays.

- What are the advantages of using binary candle?
- Why binary candle is a good idea with the advancement of age?
- What are the disadvantages of using binary candle? How will you overcome these disadvantages?

Whose cake is this?

Who does this cake belong to? There may be confusion about it. Write a detailed description about the confusion. Write a conclusion about the owner of the cake. Write its cause also. There is more than one possible explanation.

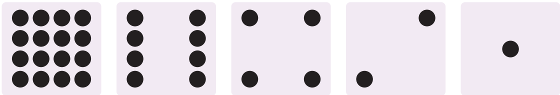
Code is for letter using binary expression

Can we send coded message to each other using letters with numbers. How many letters are there in English alphabet? Let's count together using letter cards. How can we express the letters using numbers?

We can express numbers using binary system also. What is the maximum number possible to express with this? Here we will assume A for 1, B for 2. (15)

How will we express larger number from this? What will be the number of dots of the next card? (16) (Adding a card)

Let's arrange the cards sequentially (16, 8, 4, 2, 1)



Now let's count the number "no, yes, no, no, no" using cards. How many dots shall we get? (Yes means the number is 8 for card with eight dots). What letter is for the number 8? ("H")

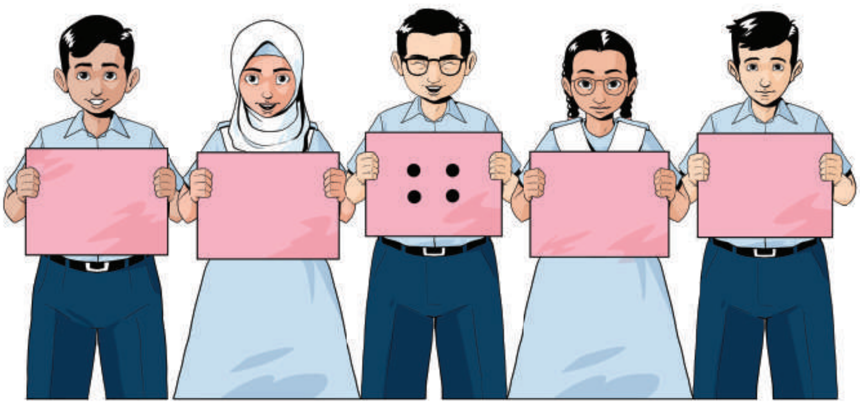
Now let's take next number. "no, yes, no, no, yes" (9). What letter is for the number 9? ("I" which can be written next to H.)

Full message is "HI".

Let's do the work how "DAD" can be written in binary code.

How can we do it?

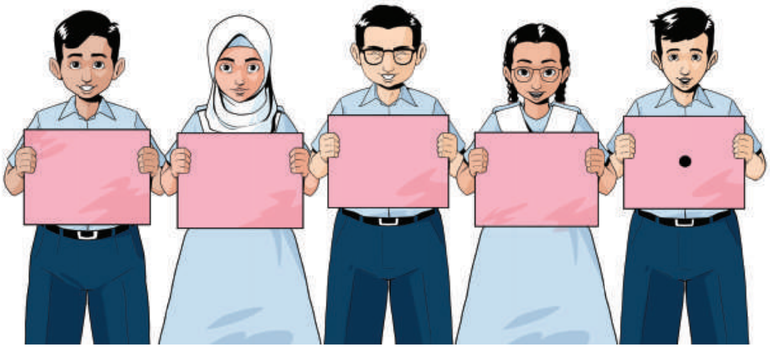
How can we make 4 using binary code?



off off on off off

A is the first letter.

How can we write 1 using binary code?



off off off off on

Oh ! However, we have written binary code of D. We can use it again. Method of use of previous work in computer science is always found out. This is a much faster way of working.

Now,let's transform a name into the binary code. Try to transform the words 'MATHEMATICS', 'BINARY', 'RAMANUJAN' into the binary code.

Individual task:

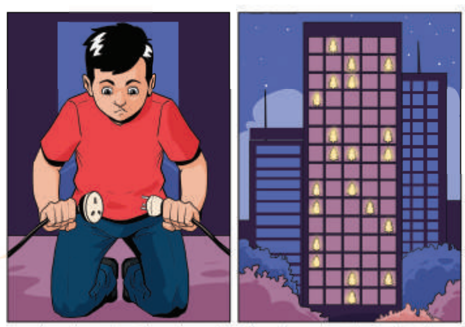
1. Necklace of binary name

Make a necklace using 5 bits binary.

Choose the colour of 1 and the colour of 0. Computer does not need to know when the new letter arrives. Because Computer knows the rule that every fifth bit is a new letter. Bit of lowest value will be on the right of every fifth group.

2. Binary to save lives

Dipu is trapped on the top floor of a departmental store. He is thinking what to do. He screams for help but no one is around. He sees a man across the street working with computer until late night. Since the language of computer is binary, so Dipu tried to convince the man by flashing the lights on and off with the binary code. What had Dipu written on the window?



1	2	3	4	5	6	7	8	9	10	11	12	13
a	b	c	d	e	f	g	h	i	j	k	l	m
14	15	16	17	18	19	20	21	22	23	24	25	26
n	o	p	q	r	s	t	u	v	w	x	y	z

Let's measure a circle

Circle

Observe the following figures. Everyday we see and use these kinds of things. Also we've made and played with these kinds of things in our childhood, isn't it?

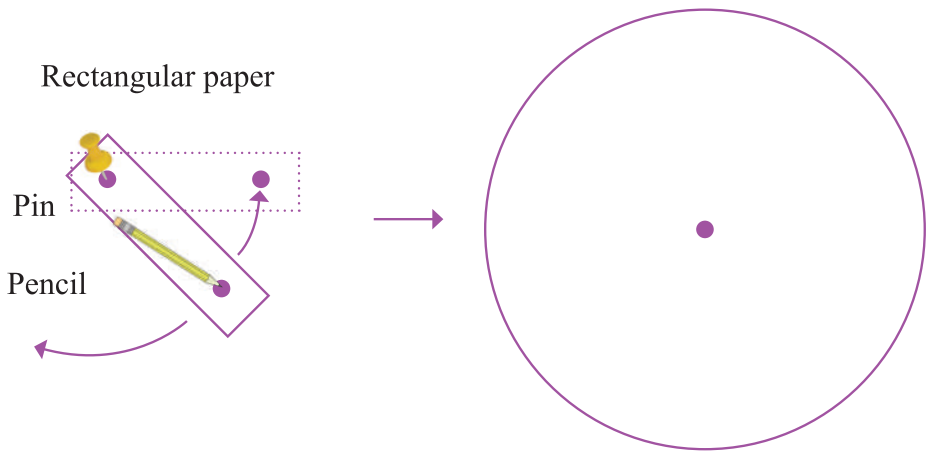
In every picture, it is seen that there is the same geometrical shape. Think it, what can we call these geometrical shapes? Yes, your thinking is right. The geometrical shape is circular.

Group work: Competition on writing names of circular things. Time: 5 minutes. Everybody of the group will write the names of circular things in his own exercise book. S/He who writes most names is the winner.

We make circle using rectangular paper

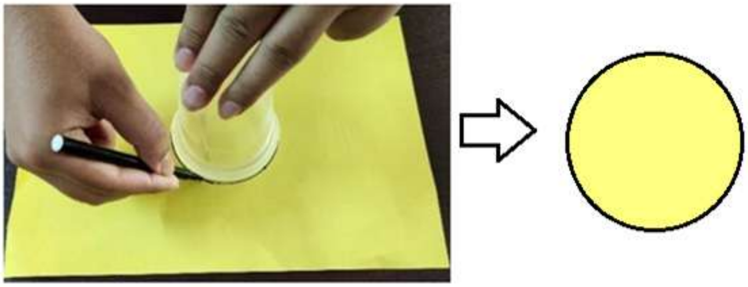
We collect a pin, a pencil, a rectangular paper with two small holes. Now using these, we draw a curve according to the following figure. If we rotate the pencil one round, then what shape will be made?

If we rotate the pencil one round, then we will get a beautiful circular shape. This circular shape is called circle.



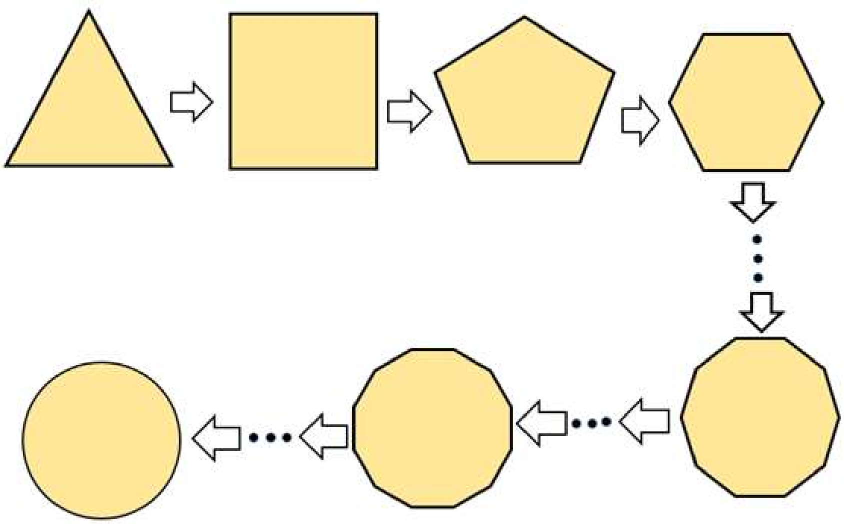
Let's draw circle cutting paper

Put a cup or plate or glass upside down on a white or coloured paper. Pressing it with the body weight with one hand, draw a line surrounding the plate or glass with a pen or thin pencil using another hand. Now if we remove the plate or glass from the figure, we will see a circular closed curve.



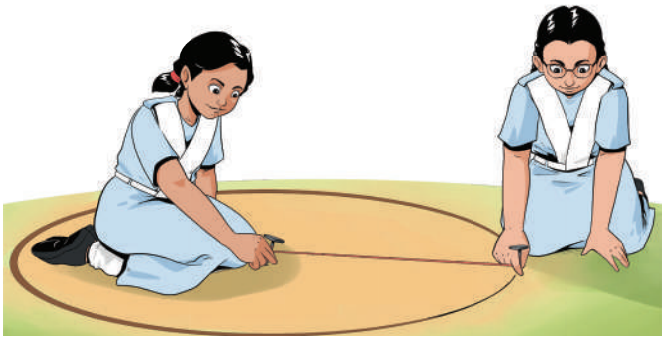
We call the circular closed curve drawing in yellow paper a circle. There is no vertex in a circle. Can you tell why the circle has no vertex?

We know, a triangle has three vertices, a quadrilateral has four vertices. Isn't it? Similarly, a pentagon has five, a hexagon has six etc. That is, the number of vertices will be the same as are the sides in a polygon. If there is infinite number of sides in a polygon the vertices will also be infinite. Then the sides of the polygon transforms into a closed curve or a circle. This will be clearer if we observe the figure below carefully.



Let's make circle using rope and nail on the ground

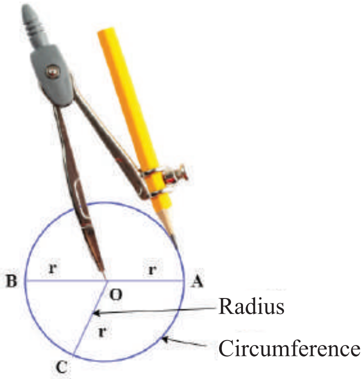
Disha decides to draw a circle on the ground using rope and pin. She ties the two ends of the rope to two nails. Now she tells her friend Mita to press a nail on one end in the ground. Pulling the pin tied on the other end of the rope around, Disha draws a circle. The circle drawn by them is like the following picture:



Group work: Like Disha, make circle on the ground using rope of different lengths dividing in small groups. Give name to the groups. Observe the circle drawn by each group and write answer to the following questions in your exercise book.

- Which group made the smallest circle and in metre, what was the length of the rope used by them?
- Which group made the largest circle and in metre, what was the length of the rope used by them?
- If the length of the rope is big, then how will be the size of the circle? Explain logically.

The circles of such drawing may not be perfect. We can draw a perfect circle using pencil compass. There, if you rotate the pencil attached to one side of compass pressing the pin of compass on the paper, it will make a circle as shown in the figure.



In this case, the point where you have pressed the compass pin is the centre. So, the point o in the above figure is the centre and the curve which bounded the centre as a circle is called circumference. Now let's find out the distance from o to the closed curve drawn in the figure, i.e. the circle. To determine this distance, draw line segment from the centre to some points A, B, C lying on the closed curve. Now determine the distance OA, OB and OC using scale. What have you observed? Are the distances equal? Each of these line segments is radius of the circle drawn by you. So, we can say, closed curve or any point upon the circle is equal distance from the centre of the circle and all radii are mutually equal.

Let's measure radius of a circle

When you draw a circle on your exercise book using pencil compass, then you can easily find out the centre and determine the radius. However, we see many circular shapes around us, big or small, which have no marked centre. We cannot determine their radius easily. In such cases, radius of a circular body can be determined following alternative way.

Pair work: Determine radius of a cork of bottle.

Every student of the pair will collect at least three corks of the same measurement according to the instruction of teacher. Then they will complete the following steps:

Step 1

Arrange the corks one after another on the paper so that they are touching each other. Attach two sticks on both sides to understand that the sticks are parallel like the picture.

Step 2

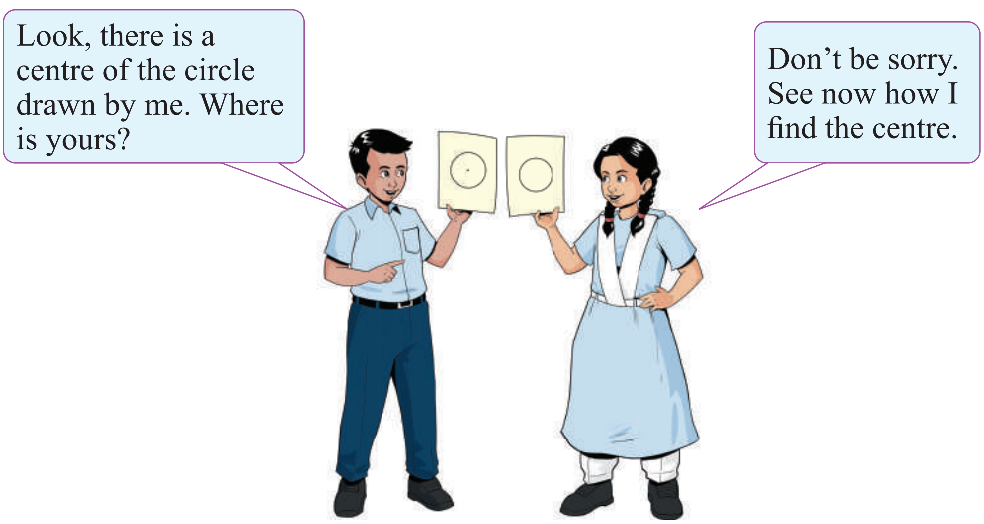
Now, using a scale, determine the length from starting point of arranging the corks to the ending point. Write down the measurement in Exercise book. You may take help of one side of a book or a stick to ensure that the corks are straight.

Step 3

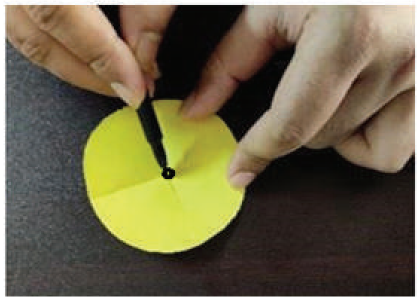
If we divide the measurement from step 2 by the number of corks we shall get the diameter of each cork. Again length of radius is half of the diameter of each cork.

Determination of the centre of a circle

You can draw circle easily using circular things you use in your daily life. But it is not easy to identify the centre. Isn't it? Have you ever thought why that is? Let's find out some way to determine the centre of any circle. In the meantime, Samir and Mira have found out two ways to determine centre of a circle. Now you have to find out whether there is any other way to do that.



Mira folded the circular paper two times, got four equal parts as in the figure below and identified the centre at the intersection point of two folds. Then drew a line following a fold and got the diameter and radius.



Individual task: Each of you will draw and cut a circle like Mira and find out the centre. You may use cup or glass or any other thing instead of bangle. You may find the centre using any other method too.

Stabilization of body

Mathematics teacher Rafiq sir wanted to know from Mira, why the centre is required in a circle? Mira could not answer instantly. Sir said that there was no problem if she had not known. He also proposed that they would play an interesting game to know the answer of this question. The game is:

You have to hold your food dish or round shape wheel at the end of your finger like the figure below. It would be enough to continue holding it at least 10 seconds the first time. Don't let it fall before that.

"What! Has it fallen before 10 seconds? Try again."

Mira found the centre of the dish after trying some times and was able to keep the dish at the tip of finger more than 10 seconds. You surely remember the race competition carrying pipkin (মাটির হাঁড়ী) on the head in the annual sports. Think whether there is a relation between that game and the above game.

Individual task: Each of you may try once like Mira.

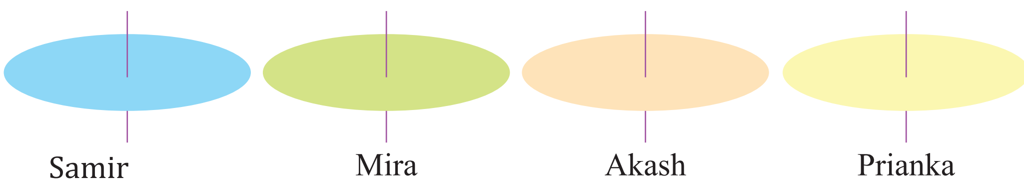
Let's cut paper and make top

Let us know why the centre of the circle is necessary by doing another task. We shall make tops with paper and examine whose top rotates at which speed.

Group work: Some groups of four are made according to the instruction of Rafiq sir. Samir, Mira, Akash and Prianka are the members of Shapla group. Sir said to the students,

- First take a cardboard or any other hard paper.
- Draw circle cutting paper. Now cut the circular area with scissors
- Insert a matchstick through big size circular paper. Now, everybody has his own top.

Samir, Mira, Akash and Prianka made four tops with different radii. The tops are as follows:



From the pictures can you say whose top will rotate the highest time?

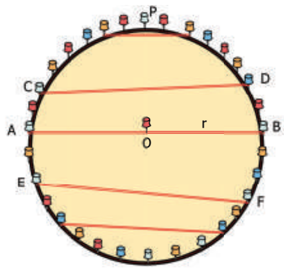
You make your own tops and spin them. Can you tell which position of the matchstick in the circular top will ensure the maximum time of spinning? Discuss in your group why that will happen.

Let’s learn about Chord and Arc of a circle

Group work: Draw a circle on paper following the figure. Then fix some pins on the circle. Keep in mind that two pins should be set on two ends of diameter of the circle. Draw the diameter and chord with rubber like the figure.

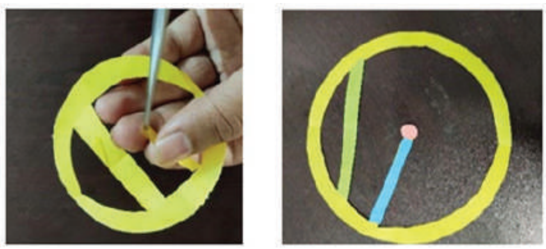
Identify by drawing a point at the root of the pin. Then discuss about radius, diameter, chord, minor arc, major arc, semicircle and all other components of circle amongst you. Measure radius, diameter, chord, arc using scale and thread and write them on exercise book. Now find answers to the following questions.

- What is the relationship between radius and diameter of a circle?
- Which is the greatest chord of a circle?
- What do we call the greatest chord?
- Diameter of a circle bisects the circle into two parts. How about their lengths?
- What do we call each of the two arcs created by diameter of a circle?



Individual task

1. Like the following figure cut a paper circle with centre, radius, chord and circumference.



2. Draw some circles of different measurements with the help of pencil compass on your exercise book. Identify the centres of circles. Taking some points in different places of the circles draw line segments from centre to the points. Draw chord through the centre or diameter of each circle. Now make a table or chart like the one below on your exercise book. Fill up the table with measurements of radius and chord through the centre or diameter of each circle and discuss the results with classmates.

Circle	Length from centre to circle or radius	Length of chord passing through the centre or radius	Explanation of the relationship between radius and chord passing through the centre and diameter of circles based on observations
1.			
2.			
3.			
4			

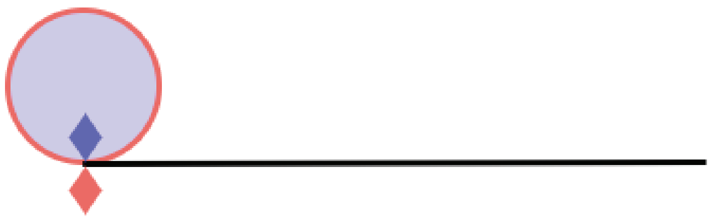
3. Make five circles cutting paper with radius of 3 centimetres. Arrange the circles as figure below; make the shape of English letter W adding the centres of the circles. Now determine the radius from

A to B. How many circles can you arrange surrounding the circle with centre C?

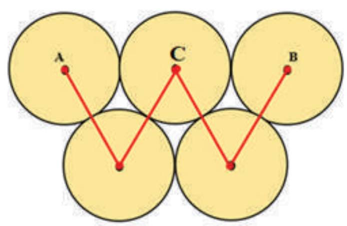
Determination of the length of circumference of circle

You have already known that the full length of circle is called circumference of the circle. Since circle is not a straight line, length of circumference of circle cannot be measured with the help of ruler. You can apply the following method to determine the length of circumference. Beside this, if you want, you can also apply another method to determine the length of circumference of circle.

- Cut a circle along its circumference after drawing it on the white page of an old calendar.



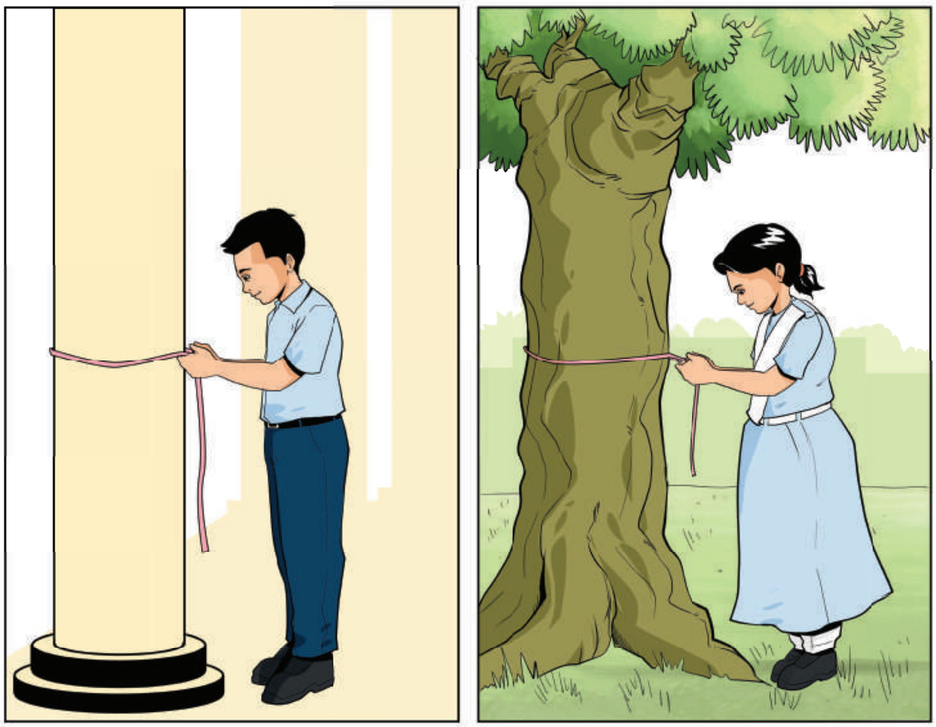
- Identify a point on the circumference.
- Draw a straight line segment setting a scale in another paper.
- Put the circular paper or card vertically on line segment drawn on paper so that the point marked on the circumference meets with the one end of line segment.



- Now roll the card along the line segment until the marked point of circumference meets the line segment again.
- Identify the point of contact and determine the length from the end point of line segment to that point using scale.
- This measurement is the length of circumference of circular card cut by you.

So we can say that if the circular card rotates one round fully, it crosses the length equal to the length of its circumference. At the time of riding your bicycle to school the two wheels of bicycle rotate again and again. Thus it crosses the length equal to the length of circumference again and again and you reach the school.

But if you are asked to determine the thickness of the round pillars of your school building or to determine the diameter of the trees of the garden of school, how will you measure them? It is not possible to rotate the pillar or trees over the line .We have to think about alternative way.



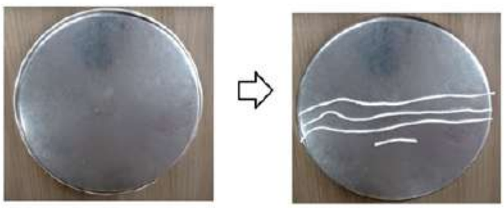
The thickness may be measured easily using tape; but what about their diameters?

Tightly tie two straight sticks with rope on both sides of the pillar. Now measure the length between the two sticks using scale or tape. The distance which is measured, is the diameter of the circular pillar.

Ratio of circumference and diameter

We have learnt how we can measure circumference or diameter of a circle. Now each of us can do the following task in our own house to know the relation between circumference and diameter of a circle.

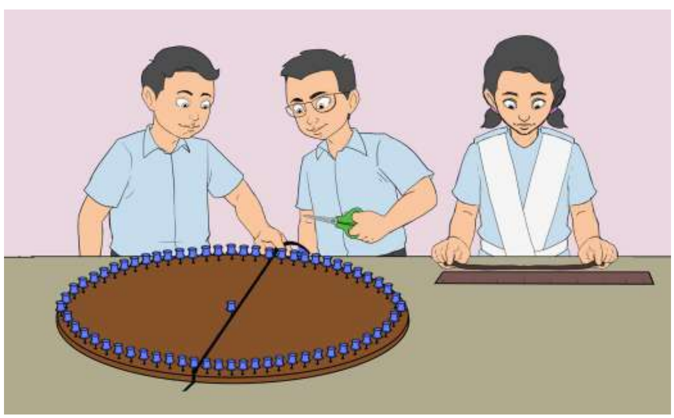
Many of us eat bread at breakfast and bread is usually circular, isn't it? Cut a thin thread rounding the outer circle of bread. The length is equal to circumference. Now cut the thread along the middle of the bread. It is equal to the diameter. After cutting the thread three times with equal portion you will see that there remains a small portion. That is, we got three full diameters and a portion of diameter. We can also verify it using circular plate, cutting watermelon.



You can also do it measuring any other circular body of your house, like, open side of utensils, glass, pail etc or upper surface of circular table, pawn of carrom, ring of different sizes etc. Beside this, you can see by cutting lemon, cucumber, brinjal circularly. Next day, discuss your experience with your classmate in your classroom.

Group task: Making Pi(π) model

Make circular model with a cork sheet or any board of thick paper. Since circle is a closed curve, it is not possible to measure directly using scale. So, tying one end of thread or thin rope with a pin on the circle, rotate it about the circle so that the thread touches the end of thread tied in pin (like the figure below). Mark along the touching point of thread and cut it with scissors or blade. Now measure the cut portion of thread using scale and write down it on your exercise book. Now measure the diameter of the circle.



Every member of the group, follow instructions and complete the task making circular areas of different radii. Draw a table like below on your exercise book. Measure the areas and note down the measurements beside your names.

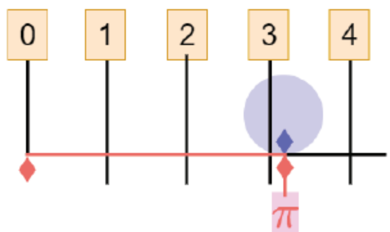
Name	Radius of circle	Diameter of circle	Circumference of circle	Ratio of circumfer-ence and diameter
Nilima				
Shahed				
Ranjana				
Pratik				
…….				

It seems that you are astonished watching the result of the table. Probably you have thought, each of you has taken circular area of different radius and also measurement of diameter and circumference are different but the results of all are almost the same. How is it possible? Discuss it in the group.

So, from the table we can decide- Ratio of circumference and diameter of any circle is constant. This constant is indicated by a Greek letter π (Pi). Greek letter π (Pi) comes from Greek circumference. Probably, William Jones at first used it in 1706.

That is, if circumference of circle is c and diameter of circle is d, then ratio of circumference and diameter is $\frac{c}{d} = \pi$ or, $c = \pi d$

Again, diameter of a circle is double of its radius; that is, if radius of circle is r, then $d = 2r$ so $c = 2\pi r$.



Mathematicians from the ancient time were trying to determine the approximate value of π. Archimedes determined approximate value 3.1419 of π determining the perimeter of regular polygon of 96 sides enclosed in a circle. Scientist Isaac Newton determined approximate value of π up to 15 digits correctly. If the radius of a circle is 1 unit, then aproximate value of π is shown as the following figure.

Indian mathematician Sreenibas Ramanujan (December 22, 1887 – April 26, 1920) correctly determined approximate value of π up to million digits after the decimal. But in the twentieth century, after invention of computer, there is a new wave to determine the approximate value of π and it is cotinuing. The reality is, π is an irrational number. [There is elaborate discussion about irrational number in the chapter of rational and irrational number.] For our daily needs of counting, approximate value of π is considered as 3.14. So we can say, circumference of circle = 3.14 $\times$ diameter of circle.

Pi day:

Lawrence N. Shaw, a physicist, curator and artist at Science Museum of San Francisco of United States of America introduced Pi day celebration. But where is the connection between 14 March and π? Why 14 March is chosen? The answer to this question is hidden on how you write your date every day. We usually write day first followed by month and finally, year; that is, 1/4/2023 means 1 April, 2023.

But some countries like USA begin with month followed by day and end with year. That means, 3/27/2023 means 27 March 2023.

Therefore, taking three digits from 3.141562, the value of Pi is written as 3/14.

Since it is written as Month/Day/Year in United States of America, so 3/14, that means 14 March is observed as 'Pi Day'. America nationally recognized 'Pi Day' in 2009.

But it seems to me, many other days beside 14 March could also be declared as 'Pi Day'.

- Which Day could be 'Pi Day' if we count as Day/ Month /Year?

Answer:

- Ok, is it possible to observe 'Pi Day' that day? What do you think?

Answer:

- If it were thought in Bangla month (Baishakh, joiustho, Asher,shrabon etc) instead of English month(January, February, March etc) then what dates in your opinion could be 'Pi Day'?

Answer:

Another interesting fact is, UNESCO declared '14 March' as International Day of Mathematics at its 40th general assembly in 2009.

You can also celebrate 'Pi Day' and 'International Day of Mathematics' as you celebrate 'Birthday of friends' in your school. You may draw picture with Pi, listen to songs about Pi, or take foods of Pi shape. You can learn more about Pi and Pi day in the link: https://www.piday.org/

Individual task:

According to the instruction, fill in the following table making it on your exercise book.

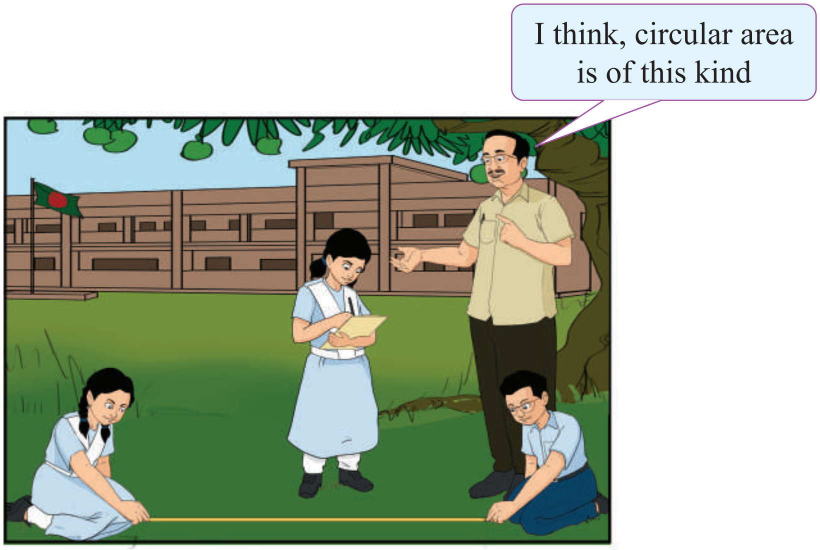
Serial no.	Radius of circle (r)	Diameter of circle (d)	Circumference of circle (c)	$\frac{c}{d}$
1	7 centimeter			
2		28 centimeter		
3			154 centimeter	
4	5.2 centimeter			
5		12 centimeter		
6			125.6 centimeter	

2. Difference between circumference and diameter of a circular park is 90 metres. Determine the radius of the park.

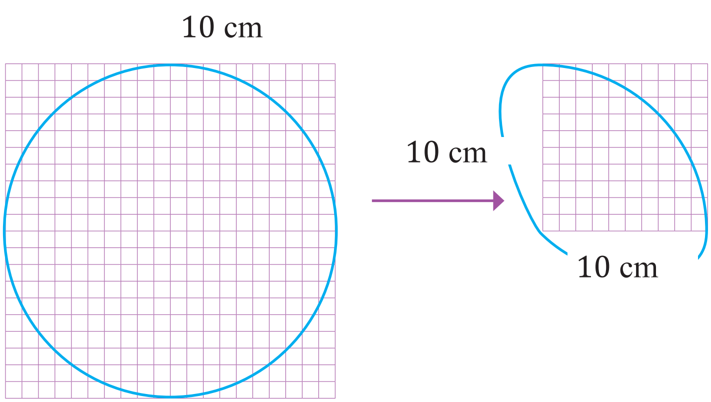
3. A car has the front wheel of diameter of is 28 centimetres and back wheel of 35 centimetres diameter. How many times will the front wheel rotate more than the back wheel in order to cross the path of 88 metres?

The area of a circle

Mira is a student of class seven. The field in front of her school is very big. Daily assembly is held in this field. Mira doesn't know why her heart is filled with joy at the gentle touch of small green grasses in the field. Mira had been thinking for a long time about planting a flower garden in the circular open space adjacent to her classroom wall next to the field. One day she told the math teacher of her class about her wish. Hearing Mira, the classmates together appealed to the teacher for gardening. The teacher became happy hearing this. He communicated the wish of the students to the respected headmaster, took permission and went to the place with the students. The place was a circular space of about 7 meters radius. He said to the students, "If we make a flower garden here, our garden will need to be nursed and fertilized. You need to know how much fertilizer is necessary per square metre. Can you tell what do we need to figure out in such that case? Is it circumference or area of the land? Almost all students together said, at first we have to find out area of the land.



Mira says, at first she will draw a circle on paper. Then cutting the circular paper, she will put it on graph paper as the following figure. Now she will determine area counting the squares of graph paper.



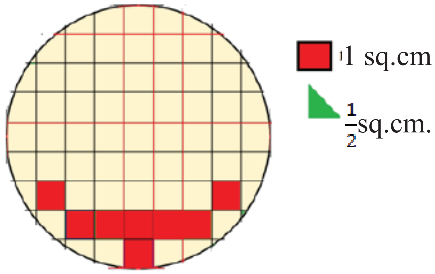
You certainly remember, we have determined area of two dimensional body using graph paper in the previous class. Isn't it? In the same way, we can also determine area of circle using graph paper.

First, divide the circular area into four equal parts.

Then one portion will be as above figure.

Now count the number of blue and red full squares. Then determine approximate area of the cut piece along the circumference, assuming each partial red coloured square as 0.5 sq cm.

Then calculation will be: the area of blue square=1× square centimetres = square centimetres



the area of red square = 1× square centimetres = square centimetres.

the area of partial red square =0.5 × square centimetres = square centimetres.

The sum of the area of full square and partial square will be the area of one fourth of the circle.

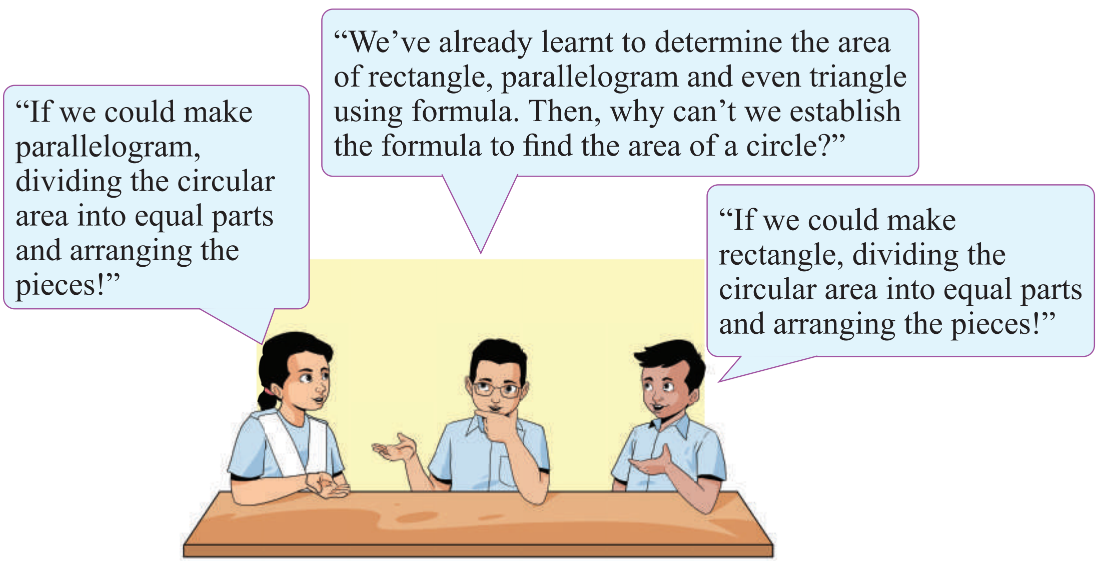
So total area of the circle =4× square centimetres.

Let us find out whether there is another way to determine the area of circle.

Group task:

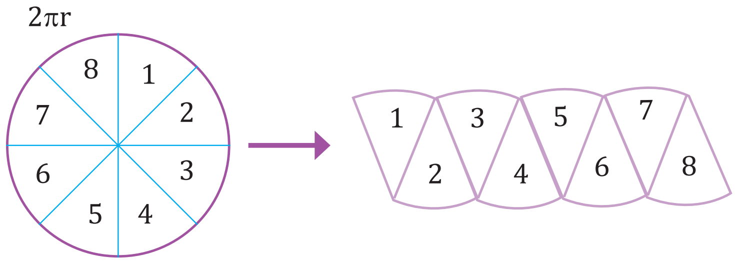
Determine area of circle measuring right angle triangle and different coloured blocks by cutting paper.

Find the formula to determine the area of circle



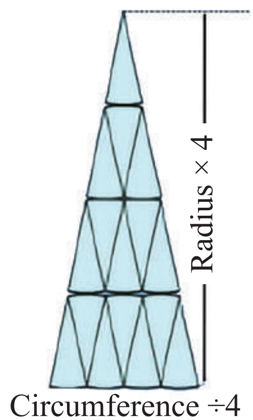
Group task:

1. Samir discussed his idea with the members of his group. Accordingly, he drew a circle on art paper or white page of back side of old calendar and cut it. Then he folded the circular area three times along the middle and cut it along the folds. Thus the circle was divided into eight equal portions. When arranged like the figure below, the circular area is transformed into a different geometric shape.

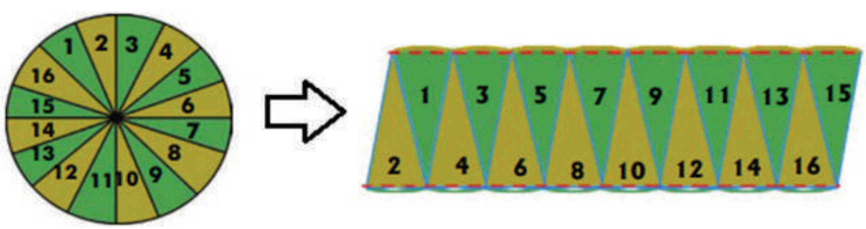


The transformed geometric figure will be like a parallelogram. Here, half of the closed curve of circle will be the base of parallelogram and radius of circle will be the height of parallelogram. Now, area can easily be determined by measuring the base and height of parallelogram. Since the parallelogram is made with the pieces of the circular area, then area of parallelogram and area of circle will be equal.

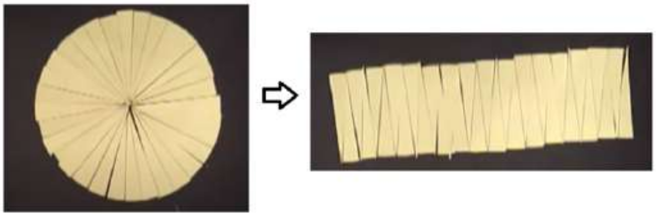
2. Sabiha divides the circular area into sixteen equal portions. When arranged, the pieces found triangular shape as in the figure beside.



3. Tarek divides the circular area into sixteen equal portions. Arranging the pieces in another way, he finds out parallelogram shape.

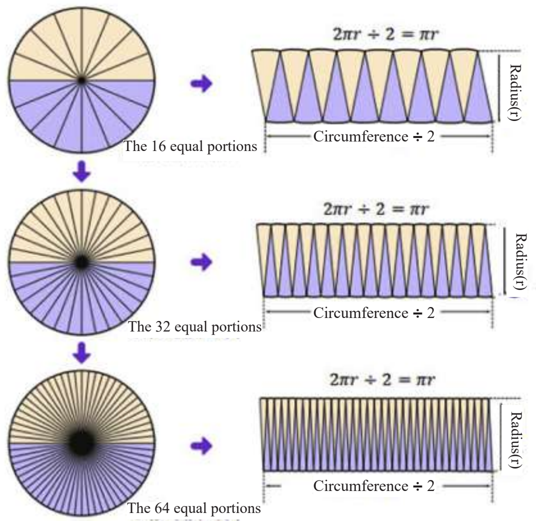


4. Mira cuts another circular area and divides it into 32 equal portions. Arranging the pieces another way, he finds the following figure:



Mira's arrangement gets the shape of rectangle. In this case, the length of rectangle will be half of circumference of circle and width will be equal to the radius of circular area.

5. If we divide any circular area into 64 or more equal portions like Samir, Sabiha, Tarek and Mira and arrange according to the above figure, then the circular area also will be like rectangle. If we observe the following figure step by step, then the conception of the fact will be clear.



Now let's find the answer to the following questions:

- Measure length (half of circumference πr) and width (radius r) of the above figure of rectangle
- Determine the area of rectangle.
- Is there any relation between area of the circle (before cutting, measure using graph paper) and area of the rectangle?

According to the above discussion, we can say-

Area of circle = Area of rectangle = length $\times$ width

= half of circumference$\times$ radius

$= \frac{1}{2} \times 2\pi r \times r$

$= \pi r^2$ sq. unit.

Pair work

(a) Draw circle with different radii. Determine the approximate area of the circle counting the least squares.

(b) Verify the area of the same circle by determining it using formula.

Individual task:

1. Each of you to draw some circles of different radii as you like. Measure the radius, diameter and circumference of the circular area. Then fill up the table determining the area by graph paper and formula.

Circle	Radius	Diameter	Circumference (using yarn or rope)	Circumference (Using using formula)	Area (Using graph paper)	Area (Using formula)	Comparison Between area found using graph paper and formula
1							
2							
3							
4							

2. Draw the following table on your exercise book and fill in the gaps by calculation

Serial No.	Radius	Diameter	Circumference of circle	Area of circle
1	12 cm	....	....	....
2	....	21 cm.	....	....
3	....	....	23 cm.	....
4	....	....	....	254.34 sq. cm.

3. Two concentric circles are shown in the side figure.

The area of right angle triangle OAB is 18 sq. m.

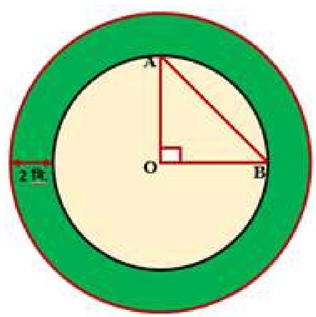
a) Determine the circumference of small circle.

b) Determine the circumference of big circle.

c) Determine the area of small circle.

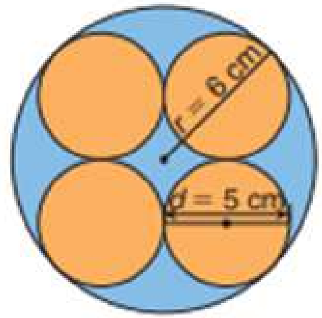
d) Determine the area of big circle.

e) Determine the area of green portion.



4. Draw a circle of radius 15 cm. on the back page of an old calendar. Now cut the circular drawing on the calendar. Cut two circular areas of radius 2.5 cm. and a rectangle of length 3.5 cm and width 2 cm. from the circle. Colour the rest as you like. Determine the area of your painted portion.

5. Diameter of a circular park is 25 metres. There is a road with 2 metres width around the park. Determine the area of the park.



6. Cut a circular area of radius 6 cm from a paper drawing. Now, cut four circular areas of radius 5 cm.

Now, colouring small circles as you like, set them inside the big circle with glue like the above figure. Then draw the following table on your exercise book and fill up the blanks.

Serial No.	Radius of circle	Diameter	circumference	Area
1.	6 centimetres			
2.		5 centimetres		
3.	The area of the large circle that is not painted			

7. Fatin went to the pizza haat with her elder sister Lamia to buy pizza. They found two packages in the price list hanging in the shop. The height of pizza in both packages is equal.

a. One pair of pizza with diameter of 35 cm. costs 300 tk.

b. Three pizzas with diameter of 30 cm. cost 350 tk.

Which package will be the best for Fatin and Lamia?

8. **Demonstration of circular bodies and detailed account project:** Divide all students in some groups. Measure radius, diameter, circumference and area of familiar circular objects of daily life. Discuss with other groups.

9. Nitu likes to embroider handkerchief, napkin, cushion or, any cloth using different yarns. She makes different designs using needle-yarn in her leisure time. The radius of the circular disc (Embroidery hoop), which is used by Nitu is 15 centimeter.

a) Determine the area of the disc.

b) Determine the area of cloth inside disc.

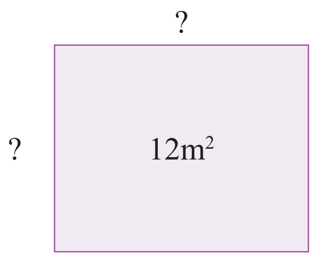
Factorization of Algebraic Expressions HCF and LCM

Factorization of Algebraic Expressions

Previously we have learnt multiplication and division of algebraic expressions, squaring expressions with two and three terms. In this episode we shall learn Factorization of Algebraic Expressions.

Each one of you take a page in your hand. Now measure the length and width of the page and determine the area of the page. You have learnt earlier that the Area of rectangular field = product of length and breadth.

Suppose the area of the rectangle is 12 cm^2. Then what can be its length and breadth?



The possibilities are:

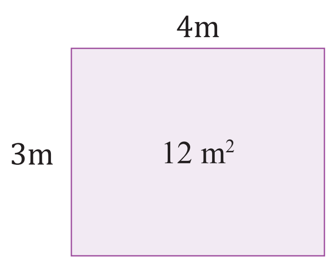
$1 \times 12 = 12$

$2 \times 6 = 12$

$3 \times 4 = 12$

Probably you are thinking which one of the above may be the answer. You are right! Each of the above alternatives may be correct. Since, if 12 is divided by each one of 1, 2, 3, 4, 6 and 12, the remainder is zero. Hence each one of 1, 2, 3, 4, 6 and 12, is a divisor or a Factor of 12.

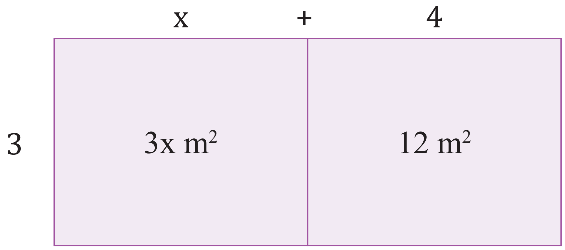
Now, suppose, we assume the two divisors or factors of 12 are 3 and 4, that is the length and breadth of a rectangle of area 12 m^2 are 4 m and 3 m respectively.



Now, if the length of the rectangle is increased by x m, then the area of the new rectangle is the new length $\times$ breadth = $(x + 4)\,3 = (3x + 12)\ m^2$.

If we ask, what are the factors of $(3x + 12)$?

Let's determine the length and breadth of the rectangle assuming the area of the rectangle is $(3x + 12)$.



Here, factors of 3 = 1, 3

Factors of 12 = 1, 2, 3, 4, 6, 12

Greatest common factor is 3

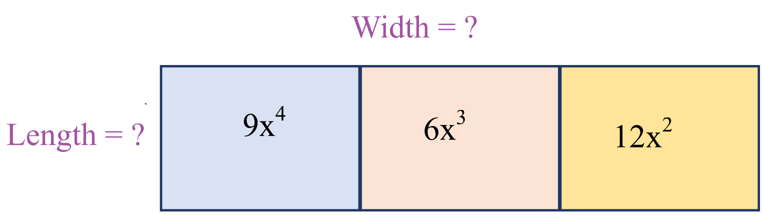
In the given diagram, we get, width = 3 m

Then length = $(x + 4)$ m

The factors of $(3x + 12)$ are 3 and $(x + 4)$ respectively.

Example 1: If the area of a rectangle is $(9x^4 + 6x^2 + 12x^2)$ m², what are its length and breadth?

Solution: Draw a diagram, using the given information that area of the rectangle is $(9x^4 + 6x^2 + 12x^2)$.



Here, factors of 9 = 1, 3, 9

Factors of 6 = 1, 2, 3, 6

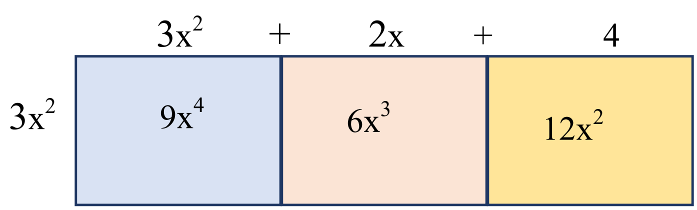
Factors of 12 = 1,2,3,4,6,12

Highest common factor is 3

Here, the highest common factor of $9x^4$, $6x^3$, $12x^2$ is $3x^2$

From the diagram we get, if breadth = $3x^2$ m

Length = $3x^2 + 2x + 4$



Hence the area is $(9x^4 + 6x^2 + 12x^2)$ m² .

Individual task: Factorise using diagrams:

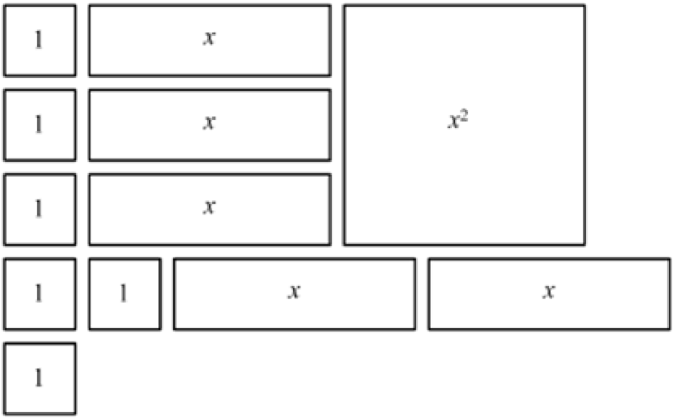
1. $20x + 4y$
2. $28a + 7b$

3. $15y - 9y^2$
4. $5a^2 b^2 - 9a^4 b^2$

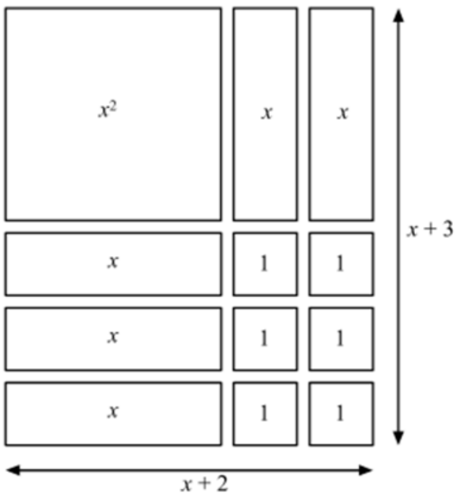
Now we'll discuss about finding factors, cutting papers.

Factorise x2 + 5x +6.

First, prepare the following blocks or models, cutting some papers and denote them with English letters.



Place the pieces of papers such that a rectangular size is made.

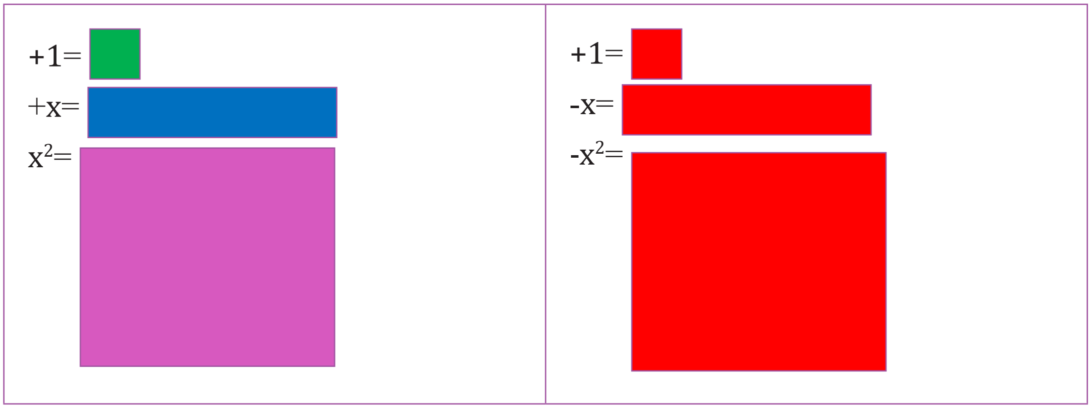


The two sides of the rectangular block are (x + 3) and (x + 2), which indicates that the factors of x2 + 5x +6 are (x+3)(x+2).

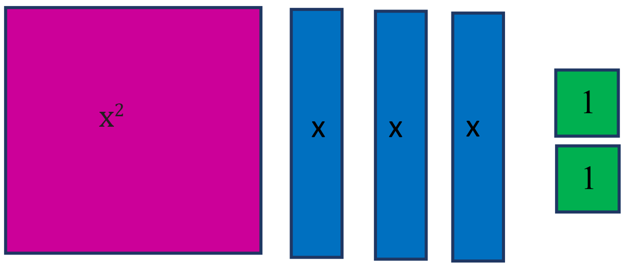
Example:

Factorise $x^2 + 3x + 2$, using the paper cutting process.

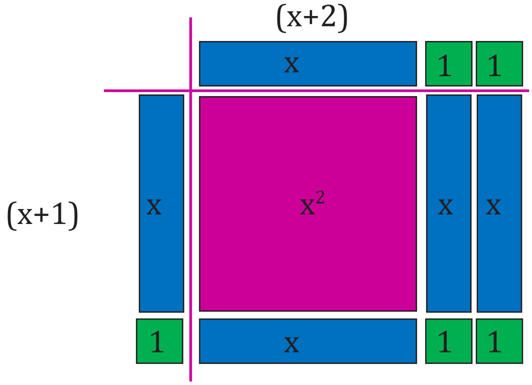
Step 1: First cut the papers and colour them as follows:



Step 2 :The following pieces of papers are necessary to factorisex2 + 3x + 2



Step 3: Try to arrange them in different shapes according to the factors, so that a rectangular shape is formed.



Step 4: Find the area of the rectangular shape using its length and breadth.

Step 5: The length and breadth of the shape denotes the factors of it.

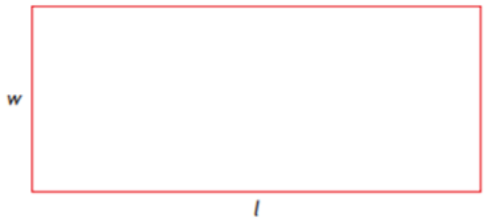
So, the factors of x2 + 3x + 2 are (x + 1) and (x + 2)

Individual task: Factorise the following according to the activities explained above.

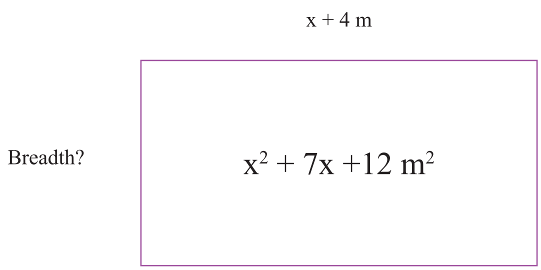
1. $x^2 + 3x + 2$	6. $x^2 + 2x + 1$
2. $x^2 - x - 2$	7. $x^2 + 5x + 6$
3. $x^2 -3x + 2$	8. $x^2 + x - 6$
4. $x^2 - 4x + 4$	9. $x^2 - 5x + 6$
5. $x^2 -2x + 1$	10. $x^2 -6x + 9$

11. The breadth of a rectangle is 14xy and area is 42 xy^3, what is its length?

12. If the length of the rectangle given in the diagram is increased by 2 units and the breadth is reduced by 1 unit, then determine what change will occur in its perimeter and area.



13. If the length of a rectangle is (x + 4) m and its area is $x^2 + 7x + 12$ m^2, then what is its width?



What is the width in metre?

HCF and LCM of Algebraic Expressions

You are already familiar with the LCM and HCF in Arithmetic. In the meantime, you have learnt about square, cube, factorisation, multiplication and division of Algebraic expressions. In this chapter, we shall learn to find the LCM and HCF of algebraic expressions.

Let us first think about the shapes of two playgrounds. Let the length and width of the first playground be x and y respectively and the length and width of the second playground be x and z respectively. Now can you tell what the area of which playground is?

Let us look at the two playgrounds in the diagram:

x y x y	x z x z
Can you tell what the area of this field is? Here length × width = area xy	What is the area of this field? Here length × width = area x z
Here, each of x and y is a factor or divisor or a multiplier, because, xy is divisible by x or y or xy, without having a remainder. And xy is multiple of x or y or xy	Here, each of x and z is a factor or divisor or a multiplier and xz is multiple of x and z.

Observe that the lengths of both the playgrounds are same. Can you tell which is the term that exists in both the areas? Yes, it is the term x that exists in both the areas. So, what can we name this term? We can say x is the common factor in both the areas of the playground, that is in xy and xz.

Common Multiplier or Common Factor: If two or more Algebraic Expressions are completely divisible by some other number or expression, then the latter is called the **Common Multiplier** or **Common Factor** of the Algebraic Expressions.

Highest Common Factor or H.C.F. : Product of all the prime Common Factors of two or more Algebraic Expressions is called the **Highest Common Factor or H.C.F.** of the Algebraic Expressions mentioned.

Example 1: Determine the Highest Common factor or H.C.F. of: xyz, 5x, 3xp.

Solution: First find the H.C.F. of the numerical coefficients of the given expressions. Here the numerical coefficients of xyz, 5x, 3xp are 1, 5 and 3, whose H.C.F. is 1.

- Now find the prime factors/multipliers of the three expressions given

The factors of xyz are x, y, z respectively

The factors of 5x are 5, x respectively

The factors of 3xp are 3, x, p respectively

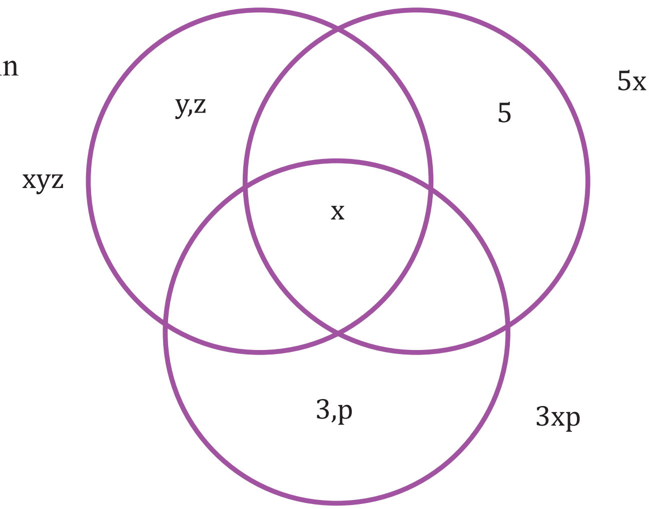
- Identify the common factor/factors from the three given prime factorization

$x\,y\,z = x\,.\,y\,.\,z$

$5\,x = 5\,.\,x$

$3\,x\,p = 3\,.\,x\,.\,p$

- Now present the factors in three circles



H.C.F. of the expressions = x

And L.C.M. =(y.z).(x).(5).(3.p)

= 15xyzp

Individual Task:

1. The Algebraic Expressions with which the H.C.F. xis formed with, can we divide those expressions by the H.C.F. x ?
2. The Algebraic Expressions with which the L.C.M. 15 xyzp is formed with, can we divide the L.C.M. 15 xyzp with those Algebraic expressions? Explain.

Example 2: Find the H.C.F. of 8 x^2yz^2 and 10 $x^3y^2z^3$.

Solution: Find the H.C.F. of the numerical coefficients of the given expressions. The numerical coefficients of 8 x^2yz^2 and 10 $x^3y^2z^3$are 8 and 10 respectively, whose H.C.F. is 2.

We find the prime factors of the two expressions 8 x^2yz^2 and 10 $x^3y^2z^3$

$8x^2yz^2$ = 2.2.2.x.x.y.z.z

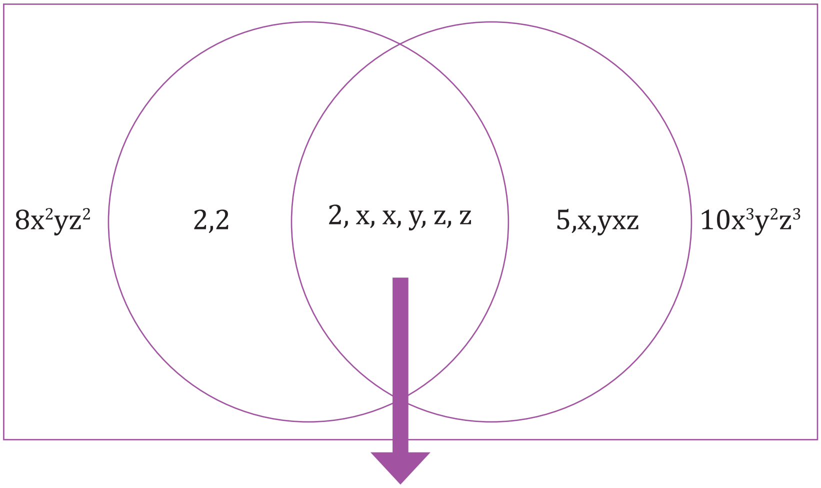
$10x^3y^2z^3 = 2.5.x.x.x.y.y.z.z.z$

- Identify the common factors from the prime factorisation of the two expressions

$8\,x^2yz^2 = 2.2.2.x.x.y.z.z$

$10x^3y^2z^3 = 2.5.x.x.x.y.y.z.z.z$

- Now represent the factors in two circles



Common factors/multipliers are on both circles

Hence, H.C.F. $=2\,x^2yz^2$

And L.C.M. = $(2.2)(\,2.x.x.y.z.z\,)(5.x.y.z) = \;40x^3y^2z^3$

Rules to find H.C.F.

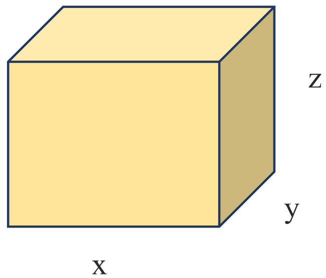
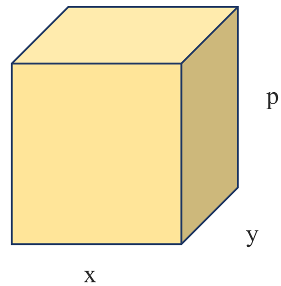
1. Have to find the H.C.F. of the numerical coefficients of the given expressions using the rules of Arithmetic.
2. Determine the prime factorization of the Algebraic expressions.
3. H.C.F. is the product of the H.C.F. of the numerical coefficients and the common factors of the Algebraic Expressions.

Task: Find the H.C.F.

1. $3x^3y^2$, $2x^2y^3$ 2. $3xy$, $6x^2y$, $9xy^2$

3. $(x^2 - 25)$, $(x - 5)^2$ 4. $x^2 + 9$, $x^2 + 7x + 12$, $3x + 9$

Now we think about the volume of two boxes. The length, width and height of the first box are x metre, y metre, and z metre respectively and the length, width and height of the second box are x metre, y metre and p metre. Can you now tell what are the volumes of the two boxes?

	
What is the **volume of the first box**? Here length x width x height = **volume** x.y.z= xyz	What is the **volume of the second box**? Here length x width x height = **volume** x.y.p= xyp
Here each one of x, y and z is a factor or divisor or a multiplier because the expression xyz is completely divisible by x or y or z or xyz And xyz is a multiple of x or y or z or xyz	Here each one of x, y and p is a factor or divisor or a multiplier because the expression xyp is completely divisible by x or y or p or xyp And xyp is a multiple of x or y or p or xyp

Observe that length and width of both the **boxes** are equal. Can you now tell which are the terms common in the **volume of the boxes**? Yes, x and y are the terms that exist in the **volume** of both boxes. So what can we say about x and y? We can call them **common factors** in the **volume of both the boxes,** that is of xyz and xyp. Again, a common multiple of the two expressions xyz and xyp is xyzp, because xyzp is divisible by each one of both the expressions.

> If an expression is completely divisible by another expression, then the first expression is a multiple of the latter. For example: The expression x^3y is completely divisible by x, x^2, x^3, xy, y etc.Hence, x^3y is a multiple of x, x^2, x^3, xy, y etc.
>
> If an expression is divisible by each one of two or more expressions, then the first expression is called the common multiple of the latter two or more expressions. For example, one common multiple of xy, x^2y, xy^2 is x^2y^2, since the expression x^2y^2 is divisible by each one of the above expressions.

Rule for finding L.C.M.:

Finding **LCM (Lowest Common Multiple):** Factorise each expression. The product of the expressions, among the above factors, each one of which has the highest dimension, will be the L.C.M. of the expressions. The L.C.M. of the numerical coefficients will be the numerical coefficient of the L.C.M.

Determine the L.C.M. :

1. $3x^2 y^3$, $9x^3y^2$ and $12x^2 y^2$,
2. $3a^2 + 9$, $a^4 - 9$, and $a^4 + 16a^2 + 9$
3. $x^2 + 10x + 21$, $x^4 - 49x^2$
4. $a - 2$, $a^2 - 4$, $a^2 - a - 2$

Full form of Lowest Common Multiple or LCM: If an Algebraic Expression is completely divisible by two or more expressions, then the expression among them of lowest dimension is called the **Lowest Common Multiple or LCM** of the two or more expressions.

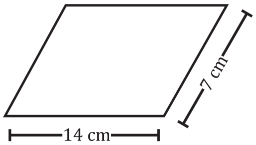
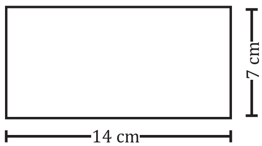
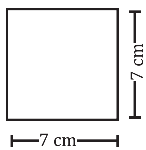
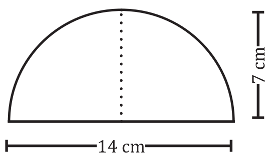
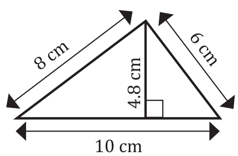
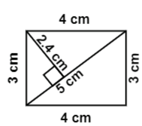
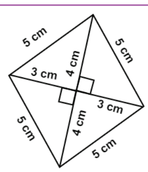
Individual Task:

Find the H.C.F.:	Find the L.C.M.:
$3a^2b^2c^2$, $6ab^2c^2$	$6a^3b^2c$, $9a^4bd^2$
$5ab^2x^2$, $10a^2by^2$	$5x^2y^2$, $10xz^3$, $15y^3z^4$
$3a^2x^2$, $6axy^2$, $9ay^2$	$2p^2xy^2$, $3pq^2$, $6pqx^2$
$16a^3x^4y$, $40a^2y^2x$, $28ax^3$	(b^2-c^2), $(b+c)^2$
a^2+ab, a^2-b^2	x^2+2x, x^2+3x+2
x^3y-xy^3, $(x-y)^2$	$9x^2-25y^2$, $15ax-25ay$
$x^2+7x+12$, $x^2+9x+20$	$x^2-3x-10$, $x^2-10x+25$
a^3-ab^2, $a^4+2a^3b+a^2b^2$	$a^2-7a+12$, a^2+a-20, $a^2+2a-15$
a^2-16, $3a+12$, a^2+5a+4	$x^2-8x+15$, x^2-25, $x^2+2x-15$
$xy-y$, x^3y-xy, x^2-2x+1	$x+5$, x^2+5x, $x^2+7x+10$

Measuring various shapes

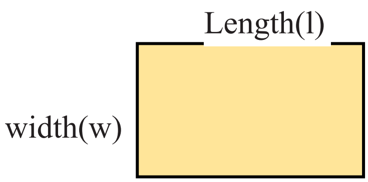
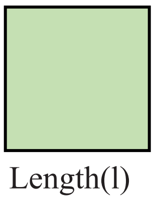
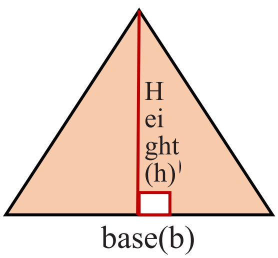
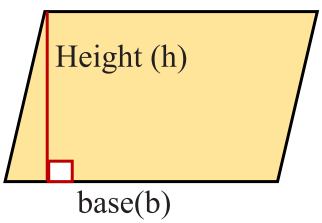
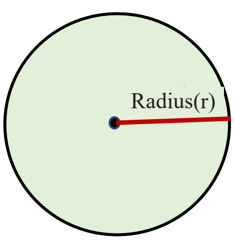
We have learnt about two dimensional plane geometric shapes. We have learnt to determine perimeter and area of triangle, parallelogram, rectangle, square and circle.

Now, let's fill in the following table. -1

Shape	Name	Perimeter	Area
	Parallelogram		
			
			
			
			
			
			

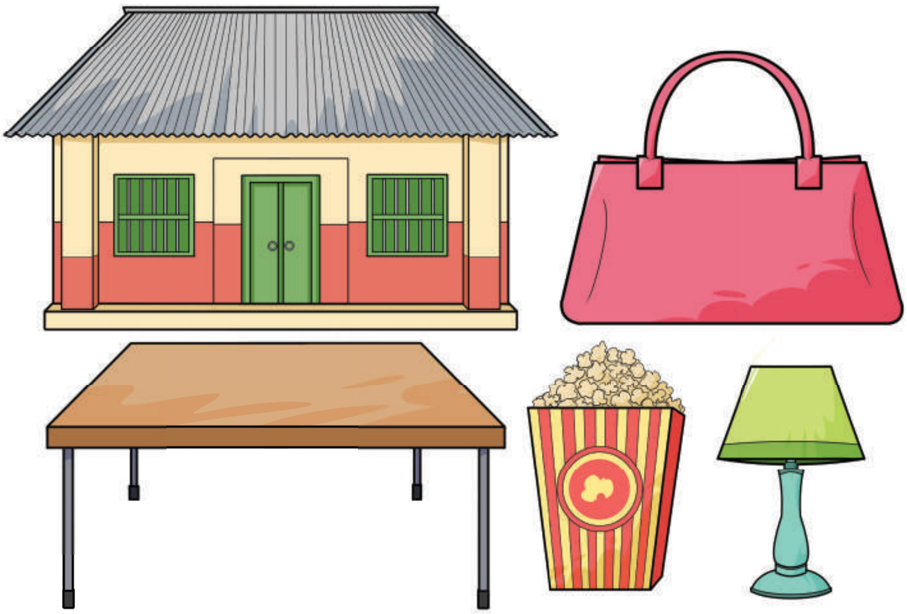
Suppose you don't know the length and height. Let's express the length and width using unknown variable instead of value.

Table-2

Shape	Name	Area	Perimeter/ circumference
			
			
			
			
			

Let’s measure area of trapezium

Salam sir teaches mathematics. One day, he said to the students in his class, “We have learnt about the shapes of rectangle, square, parallelogram, triangle, even circle. We have learnt to determine the area of them. We use many things or there are many land areas surrounding us, whose shapes are as follows”:



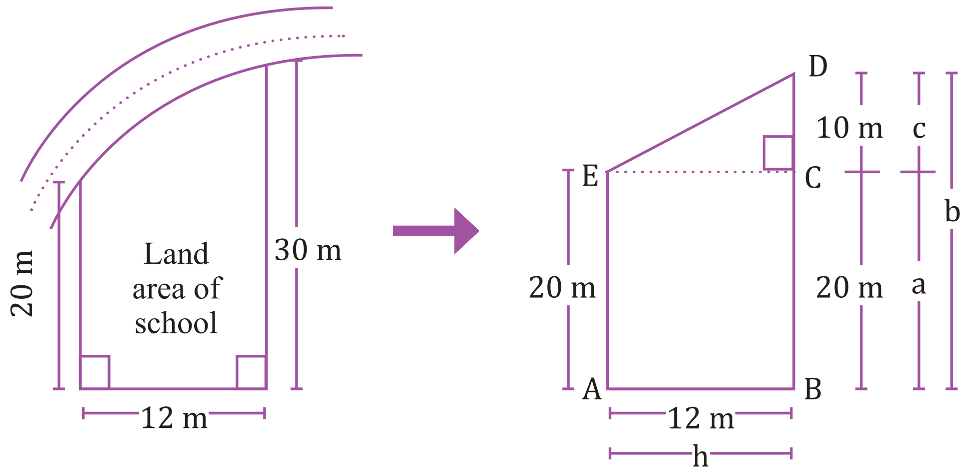
“If we watch carefully, we will see specific portions of the above pictures are in the same shape. We have learnt such shapes in the previous classes. Can you tell what we call those shapes?”

“Yes, we call such shapes trapezium.”

“Is there any similarity of the land area where the school is established, that is, the shape of the p perimeter of the land of our school with the shape of trapezium?”

“Let’s measure the trapezium shaped land of our school today.”

Salam sir takes a long measuring tape and goes to the school’s field with students. Students draw the following figure measuring the perimeter of the land of school according to his instruction. They get the quadrilateral putting A, B, D and E on the vertices of the land. Two opposite sides of the quadrilateral AE $\parallel$ B and the other two sides are nonparallel. So the quadrilateral ABDE is a trapezium. The students divide the trapezium ABDE into two parts. First part ABCE is a rectangle and second part ECD is a right angled triangle. Since the students know how to measure the area of rectangle and triangle, so they calculate the area of the land of their school in the following way.



Calculation:

(a) The area of rectangle ABCE = length × width = AE × AB= ☐ × ☐ sq.metre= ☐ sq. metre

(b) The area of triangle ECD = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × EC × CD sq. metre

= $\frac{1}{2}$ × ☐ × ☐ sq. metre = ☐ sq. metre

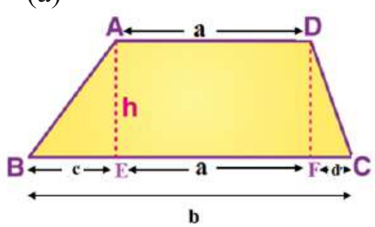
So, the area of the trapezium shaped land ABDE = The area of rectangle ABCE + The area of triangle ECD

= ☐ sq. metre + ☐ sq. metre

= ☐ sq. metre

Let's find formula to determine the area of trapezium

(a)



The area of trapezium shaped land = the area of rectangle AEFD + The area of triangle ABE + The area of triangle DFC = $\left(a.h + \frac{1}{2}h.c + \frac{1}{2}.h.d\right)$ sq. unit.

$= \left(a + \frac{c}{2} + \frac{d}{2}\right) \times h = \left(\frac{2a+c+d}{2}\right) \times h = \left(\frac{a+a+c+d}{2}\right) \times h$ sq. unit.

$= \frac{1}{2}\{a + (a + c + d)\} \times h$ sq. unit.

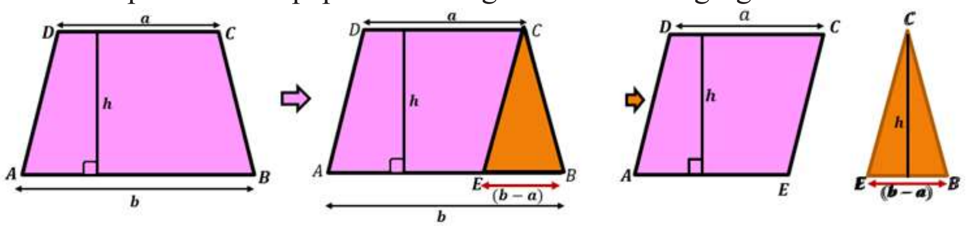
$= \frac{1}{2}(a + b) \times h$, Since $a + c + d = b$

$= \frac{1}{2}(AD + BC) \times h$ sq. unit.

$= \frac{1}{2} \times$ (Sum of two parallel sides × height) sq. unit.

Determination of area of trapezium in an alternative way

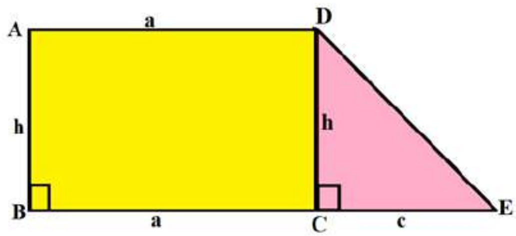
1. Draw a trapezium on a paper according to the following figure and cut it.



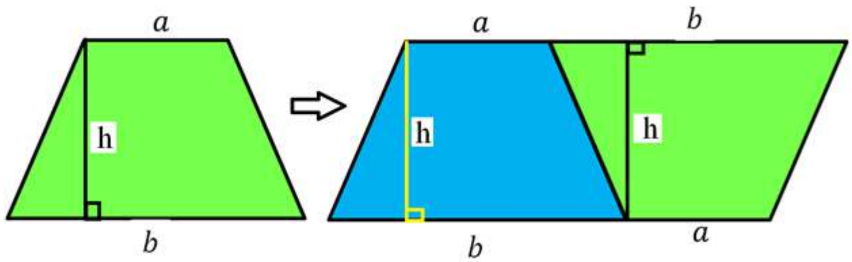
2. Measure two parallel sides and the height. Note them down on your exercise book.
3. Now, construct a parallelogram taking the equal length from larger side to the smaller side.
4. Then, cut the triangle part to separate it. So the trapezium will be divided into parallelogram and triangle.
5. You have already learnt the formula to determine the area of parallelogram and triangle. So, I hope, you will be able to determine the area of trapezium easily using the formula of parallelogram and triangle.

Pair task: Construct model, cutting paper as (a), (b), (c) in the figure below. Then determine area in more than one alternative way.

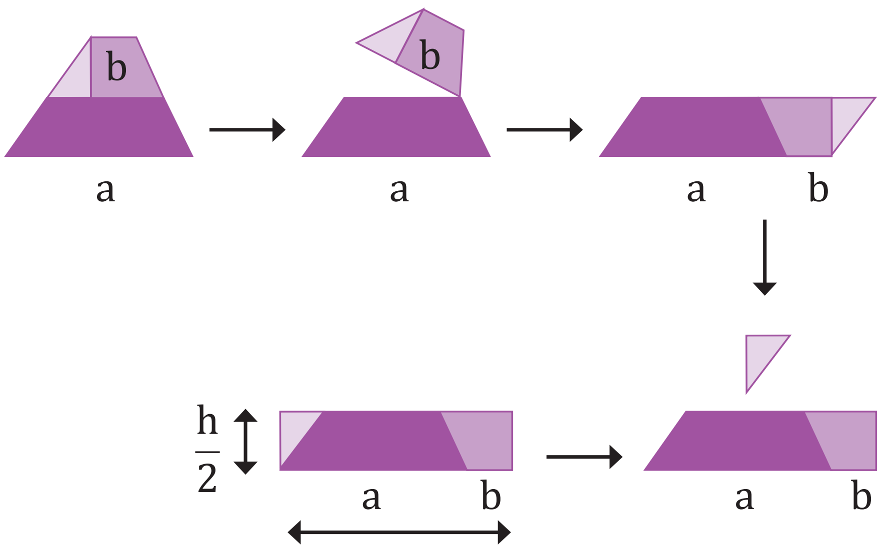
(a)



(b)

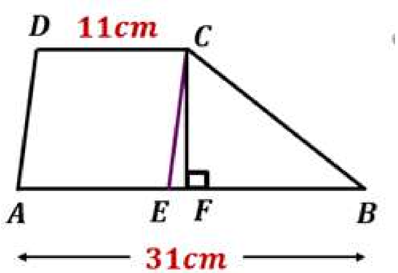


(c)



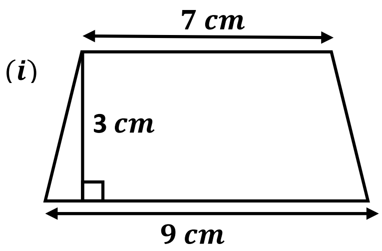
Individual task:

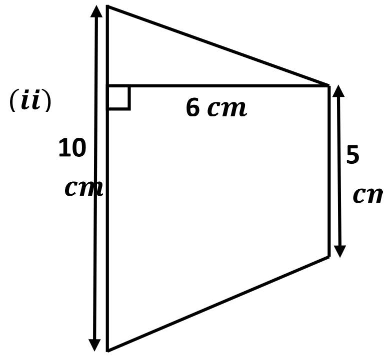
1. Draw a trapezium on a graph paper. Determine the area of the trapezium assuming every smallest square as one sq. unit and partial smallest portion as 0.5 sq. unit.
2. The difference between the lengths of parallel two sides is 8 cm. and perpendicular distance between them is 24 cm. If the area of the trapezium is 312 sq. cm. then determine the lengths of two parallel sides.
3.



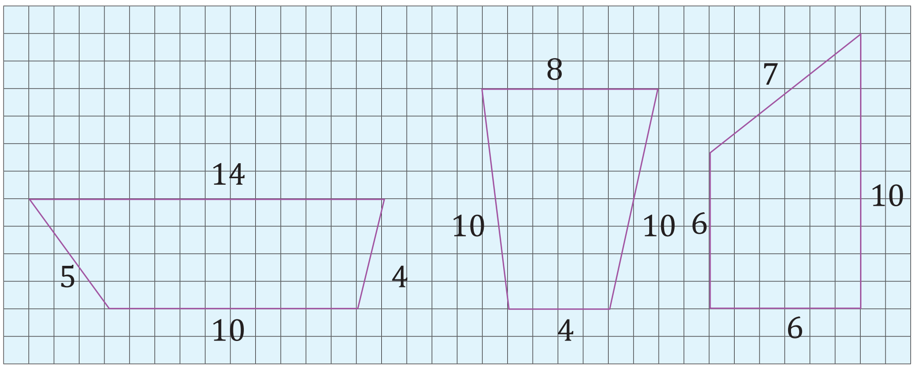
If the area of the triangle ΔBCE is 100 sq. cm. determine the area of trapezium ABCD.

4. Determine the areas of the following two trapeziums.



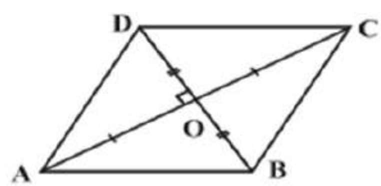


5. The areas of which trapeziums are equal but the perimeters are different?



Let's find formula to determine the area of Rhombus

Let the following figure ABCD is a rhombus, whose diagonals are AC and BD. You have surely learnt that two diagonals of rhombus bisect with each other at right angle. Then, the diagonals AC and BD bisect with each other at right angle at the point O. Again, diagonal AC divides the rhombus ABCD into two portions.



So, we can say,

The area of rhombus ABCD = The area of ΔADC + The area of ΔABC

$$= \frac{1}{2} \times AC \times OD + \frac{1}{2} \times AC \times OB$$

$$= \frac{1}{2} \times AC(OD + OB)$$

$= \frac{1}{2} \times AC \times BD$

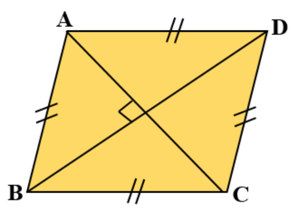
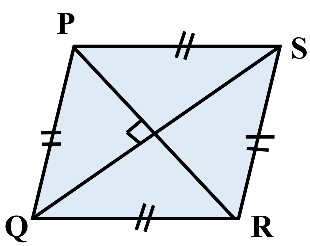
$= \frac{1}{2} \times d_1 \times d_2$; where $AC = d_1$ and $BD = d_2$

$= \frac{1}{2} \times$ product of two diagonals.

Area of rhombus = half of product of two diagonals.

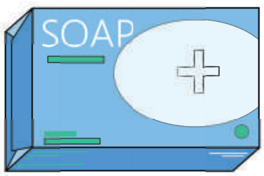
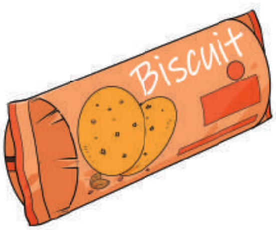
Individual task:

Fill in the table

Shape	Name	Diagonal (d_1)	Diagonal (d_2)	Area
		AC=d_1=8 cm	BD=d_2=12 cm	
		PR=6 cm		42 sq. cm.

Solids

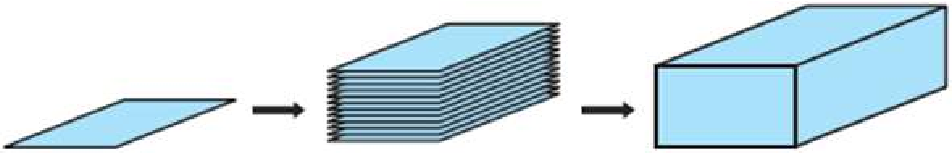
We are all more or less familiar with the following things. Isn't it? We use toothpaste, soap, biscuit, medicine and many more daily essentials. We have learnt about such shape of packet or box in the previous classes. Now, fill in the blanks of the table observing the following things carefully and draw pictures, write down the name of the shapes, sizes, number of surfaces and packets of two or three more things known to you.

Things	Name of shapes with packet	Size of each surface	Number of surfaces
			
			
			
			

You have thought about the shape of packets of different things in the above table. However, you should have some ideas about shapes of your study materials like your books, exercise books, pencils and pens. Have you observed any difference between the shapes of your mathematics book and pencil? Again, you and your friends sometimes enjoy competitions with Rubik's cube. Though the shape of this Rubik's cube is like a thick dictionary, observing carefully you can differentiate between shapes of the two things.

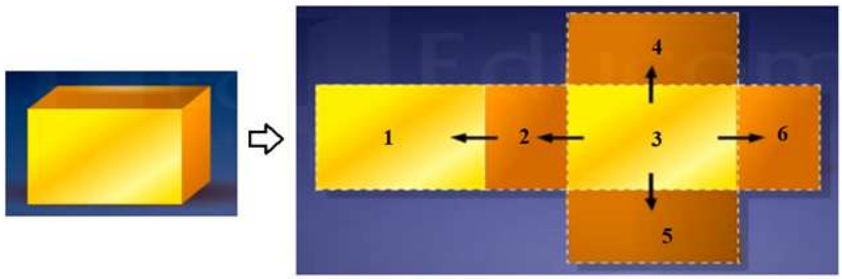
Now, let's know how your book is made and what can we call the built shape? Take some papers of equal size. A4 size paper will be better for you.

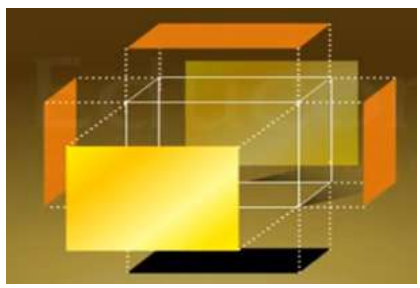
You already know that a leaf of A4 size paper is considered as two dimensional rectangle. Now putting the paper on the table, if we put some more papers on top of it, then it will be like the following picture.



So, the shape you will get at the end will be a rectangular solid. Although one leaf of paper is two dimensional (Only length and width is considered), we get another dimensional height if many papers are piled putting one atop another.

Then, we can say, there are length, width and height of a rectangular solid. That is, rectangular solid is three dimensional.





Observe the following picture. This is a box. The shape of the box is rectangular solid. We will see six surfaces if we open the surfaces of the box carefully. You may open a tissue box or toothpaste packet carefully. You will see, there are 6 surfaces, 12 edges and eight vertices in the box or packet. Again, if you see the surfaces of box like the followings, you will get three pairs of identical opposite plane surfaces.

We can determine the whole area of surfaces measuring every rectangular plane or surface of the box. Although in the previous class we learnt to figure out the total surface area of such a box taking measurements, it would be nice to have a little more practice, wouldn't it?

Group task:

Let's make a rectangular solid and measure the total surface area and volume

Let's measure the total surface area

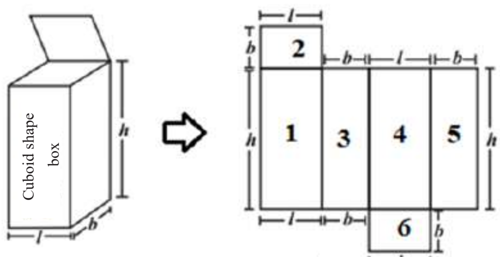
- First cut the paper and make a rectangular solid structure like the picture below.
- Cut the paper measuring equal to each surface of the structure.
- Draw small square units on each surface of paper equal to the drawing structure.

- The solid will be constructed if you attach the drawn paper with glue according to the six surfaces of the structure.
- Set the numbers 1, 2, 3, … sequentially in the small cells or rooms in each surface. Each of these rooms is a square as lengths of sides of each are equal or 1 unit. That is, they all are "unit squares". You have surely learnt, "If you divide the area of a region (like: triangular area, square, rectangular area) into squares, the area is equal to the sum of square units it is divided into." So, the sum of the small cells or rooms of all rectangular surfaces will be the whole area of the solid. Then, the greatest number written on cell or room of the surfaces will be the whole area of the solid.

Let's find the formula to determine surface area of a solid

Rectangular solid (Cuboid)

Every equal plane of rectangular cuboid is rectangular. Let's try to construct an algebraic formula to determine the surface area of a cuboid defining the length and width of its surfaces with unknown symbols as in the figure below. Let, you have cuboid shaped box. You can identify the dimensions that is, length (l) , width (b) and height (h) like the following figure.



(a) (b)

Opening the box, you get six surfaces. If you observe, you will see that each of them is rectangular. You have learnt to determine the area of rectangle, isn't it? Think, whether it is possible to determine the whole surface area of the box?

If you determine the area of six surfaces, then the surface area of the box will be the sum of these separately determined surface areas of six surfaces. That is,

Surface area of all surfaces of the box

= Area 1 + Area 2 + Area 3 + Area 4 + Area 5 + Area 6

$= (h\times l)+(l\times b)+(b\times h)+(h\times l)+(b\times h)+(l\times b)$ sq. unit.

$= (hl+lb+bh+hl+bh+lb)$ sq. unit.

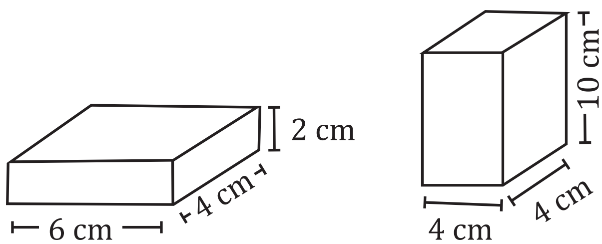
$= (2lb+2bh+2hl)$ sq.unit.

$= 2(lb+bh+hl)$ sq. unit.

So, we can tell, if a cuboid with length (l) width (b), and height (h), then whole surface area of the cuboid = (A) $= 2(lb+bh+hl)$sq. unit.

Individual task:

Determine the whole surface area of the following figure (a) and (b)



(a) (b)

Group Task:

Measure the length, width, and height of the class room. Then answer the following questions.

a. The whole surface area of classroom (Excluding door and window)

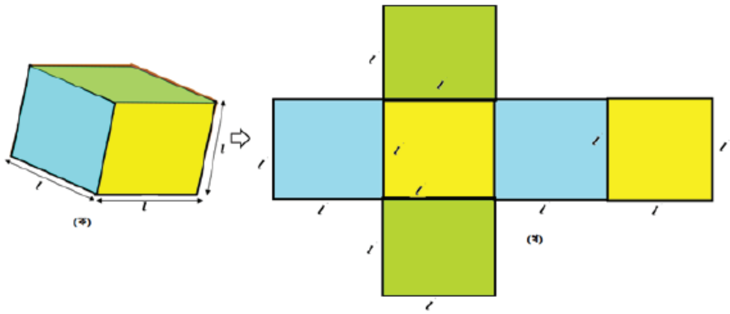
b. Area of side surfaces.

c. Prove that the whole surface area of classroom = area of side surfaces +2 × area of floor.

Cube

You have taken such a box all of whose sides are equal. That is, length of the box = width = height. What will you call such a shape of equal length, width and height?

The shape of the box will be like the figure (a). If you open the surfaces of the box, then it will be like the figure (b). Let, the edge of box is (l) unit.



(a) How will be the shape of each surface of the box?

(b) Determine the area of each surface. Are they equal with each other?

(c) Determine the whole surface area of the box.

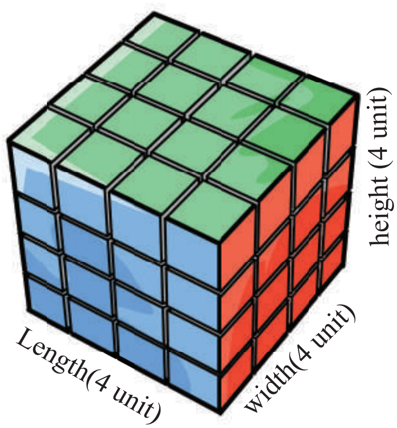
Therefore, we can tell, if the edge of a cube is l unit, whole surface area (A) = $6l^2$ sq.unit.

Individual task:

1. Minati makes two cuboid shape boxes using paper like the figure below. Which box took lesser quantity of paper to be made?
2. Rabin has a cabinet whose length, width and height are 2 metres, 1 metre and 3 metres respectively. He wants to paint outside the cabinet excluding the lower surface. If it costs 150 Tk. per square metre, then how much cost will be there?

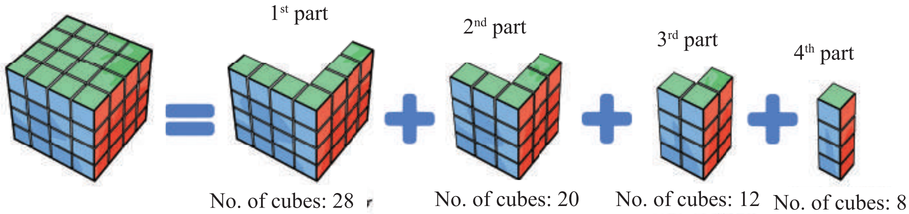
Let's find formula to determine the volume of a solid

You have already learnt the volume of a solid is three dimensional. That is, there exists length, width and height of the solid. If we can make some small boxes of length 1 unit, width 1 unit and height 1 unit and fill in the cuboid with the boxes, then we will get the volume of the solid. Since the length of the small boxes = width = height. So every box will be a cube. The volume of the solid will be how many unit cubes there are in the solid. Here unit cube is that cube whose length = width = height = 1 unit.



Now, let us give an example.

Almost all of us are acquainted with Rubik's cube. You may think, why Rubik's cube is recalled as an example? The cause is if you watch carefully, you'll find Rubik's cube is made of many unit cubes. Beside this, many of us play with it.



Here we have seen such a Rubik's cube whose length = width = height = 4 unit. We can see many cubes inside the picture. Each of them is a 'unit cube' because each side length of the small cubes inside the picture is 1 unit. So, all of them are 'unit cubes'. Now, we have to count all small cubes inside the picture one by one. The volume of the Rubik's cube will be as how many cubes there are. Now let's start counting. We will divide the Rubik's cube into some parts so that it is convenient for us to calculate and understand.

Observe the following picture carefully.

In the above figure, the cube taken by us is divided into four parts. The Rubik's cube can be formed with these four parts. We see in the figure:

The number of cubes

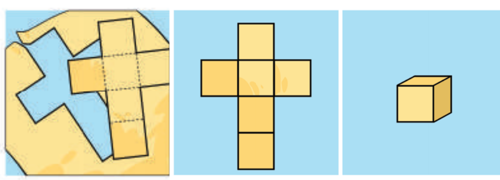
in the 1st part = 28, in the 2nd part = 20, in the 3rd part = 12 and in the 4th part =4.

So, sum of the cubes = 28+20+12+4 = 64

So, the volume of the Rubik's cube taken by us = 64 cubic units.

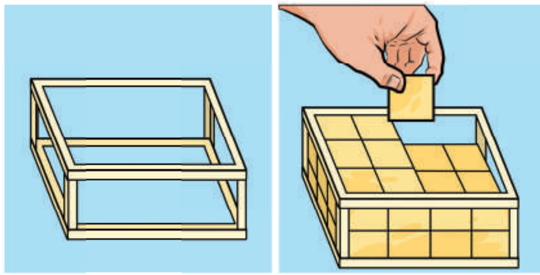
All of us have learnt about the volume of solid from above discussion. To make the concept clearer, let's cut the paper and make a box of 1 unit length, width and height. For these, we have to do the following task step by step.

- First take one leaf of paper.
- Now, cut like the figure below, a piece equal to any side of the small square of surface of the previously made solid.



- Cube shape box will be made if you fold the paper cutting along the marked line and attach the surfaces with glue or scotch tape.
- Make more such boxes as you don't know in advance, how many small boxes are necessary to fill up the rectangular solid completely.
- Now keep arranging small boxes one by one inside the rectangular solid structure as shown in the picture below.

- When the rectangular solid structure is completely filled up, count the number of small boxes. The volume of the rectangular solid will be the same units of small boxes contained inside the structure.



So, we can decide, the volume of the solid is the space bounded by the dimensions of the rectangular solid. The volume of the Rubik's cube taken by us was 64 cubic units. Again, this 64 is the product of three 4's. That means, 64 = 4×4×4 = = length × width × height.

Since in a cube, length of the cube = width = height, so it is considered each of the length, width and height as edge of the cube.

If edge of the cube is l unit , then

Volume (v) = length × width × height =l×l×l=l3cubic unit.

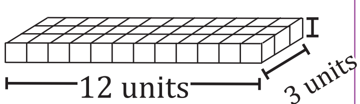
Again the rectangular solid you have made cutting paper had length of 4 units, width 3 units and height 2 units. And a total of 24 small boxes were needed to fill up that solid structure, isn't it? So think, whether there is any relation between length, width and height of the solid and 24 small boxes made by you? That is, we can tell volume of the rectangular body = length × width × height.

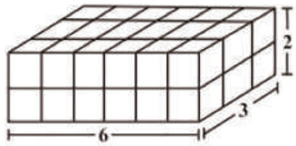
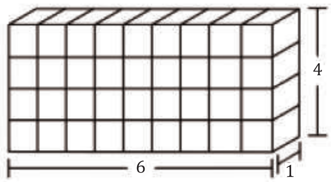
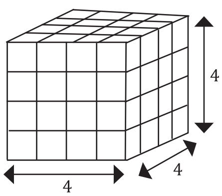
If length (l), width (b) and height (h), then

Volume = length × width × height = l × b × h cubic unit.

Individual task:

1. Fill in the following table.

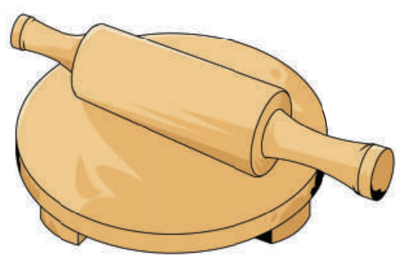
Serial no.	cuboid	Length (1)	Width (b)	n Height (h)	Whole surface area	Volume
1.		12	3	1		

2.						
3.						
4.						

2. Calculate the total surface area and volume of the math book measuring its length, width and height.
3. Three metal cubes have edges 3 cm, 4 cm and 5 cm respectively. Three cubes are melted to form a new cube. Determine the total surface area and volume of the new cube.

Cylinder

or cylinder calls to our mind two materials like the image below, right? They can be found in each of our homes. Many of us eat roti-parata (bread) especially for breakfast. And the following two things are used to make it. Can you tell which is called what?



The top material with handles is called cylinder and the bottom round object is called bread making disc. Now you have to do a job. You have to find out the radius, diameter, circumference and top floor area of the bread making disc you have at home.

Determine the area of breads (at least three) made for you. Now fill in the table below with your opinion about the area between the bread and disc.

Materials	Radius	Diameter	Circumference	Area
Disc				
bread-1				
bread-2				
bread-3				
bread-4				
bread-5				
Opinion				

In our daily life, everybody see or use various kinds of things like cylinder. You will see test tube, beaker and even tube light of classroom when you have practical classes in your school's science lab. Think, whether the shapes of these things are similar or not? It seems, many trees surrounding us are cylindrical. As for example: betel nut tree, palm tree etc. Looking at the pictures below, we can remember of many more objects of this shape.

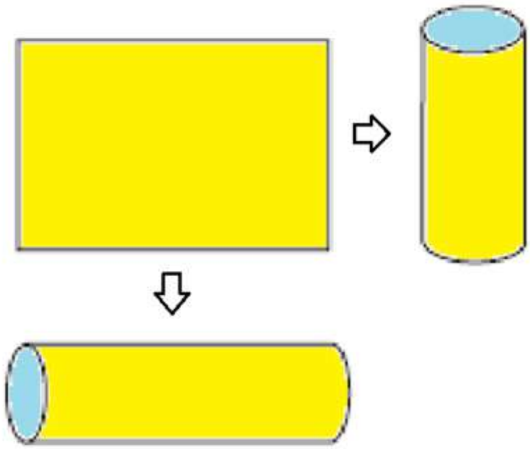
Group task:

"competition on writing the names of cylindrical objects." Time: 5 minutes. Everybody of the group will write the names of cylindrical objects on his/her own notebook.. The group who writes most names is the winner.

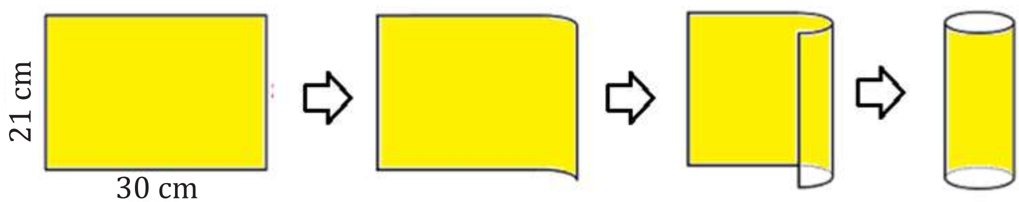
Let's make paper cylinder

So far we have learnt the names of many cylindrical objects and their uses. Now let's cut the paper and make a sample cylinder.

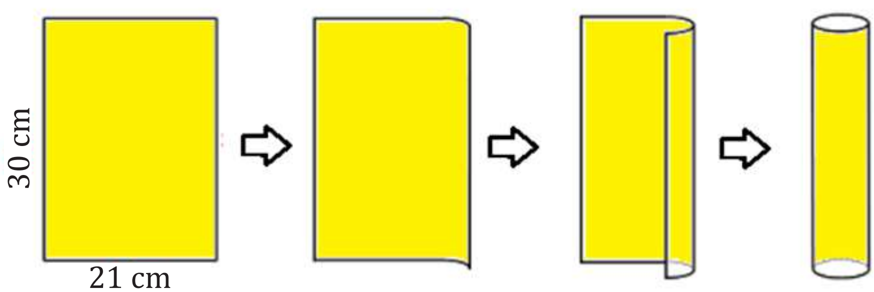
- First take a leaf of A4 size paper. If A4 size paper is not available, take any other rectangular paper.
- Two right circular cylinders can be made by turning the two ends of the paper along the length and width as shown in the figure below.



- We know that an A4 size paper is roughly 30 centimetres long and 21 centimetres wide. First roll the paper along the width and make the cylinder.



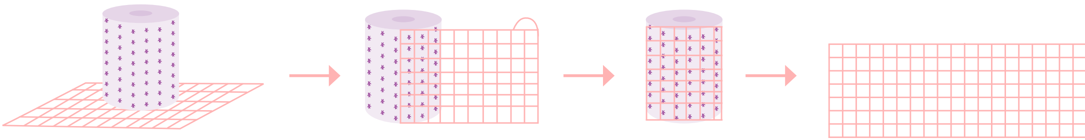
- Now in the same way, roll the paper along the length and make another cylinder.



- You have made two cylinders by rolling the same size paper lengthwise and widthwise. Now, after a little thinking, tell whether the area of the wrapped up surface or curved surface of two cylinders will be similar or different?

To know the answer to the question, first we have to know the area of wrapped up surface or curved surface of the cylindrical solid.

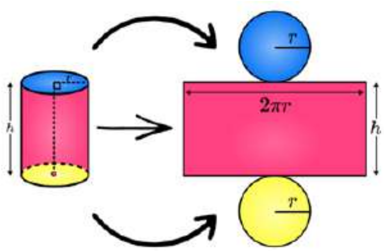
Take a cylindrical object. A piece of pipe or a battery will do. Place the battery on the graph paper and cut it in such a way that the width of the graph paper is equal to the height of the battery as shown in the figure below. Now cut the graph paper along the length in such a way that the graph paper looks like a packet.



What shape do you get after separating the graph paper from the battery? Definitely rectangular, isn't it? You must know that each of the small cells or rooms of the rectangular paper is a square as the length of each side is equal or 1 unit. That is, all of those are unit squares. Now count the number of the small cells or rooms. The sum of these rooms will be the area of this rectangular paper. That is, the area of the curved surface of the battery.

Let's find the area of curved surface of cylinder

The cylinder you made by folding the rectangular paper has two identical open mouths. These two open mouths are actually two identical circles. If the rectangular paper is rolled along the length, the length of the paper will be equal to the circumference of the circle. In that case, the width of the paper will be the height of the cylinder.



You have already learnt, if the radius of a circle is r units then the circumference $= 2\pi r$ units. So, the length of the rectangular paper will be $2\pi r$ units. Width of the paper = height of the cylinder = h units.

So the area of the curved surface of the cylinder = Area of the rectangular paper.

= length $\times$ width

$= 2\pi r \times h$ sq. unit.

$= 2\pi rh$ sq. unit.

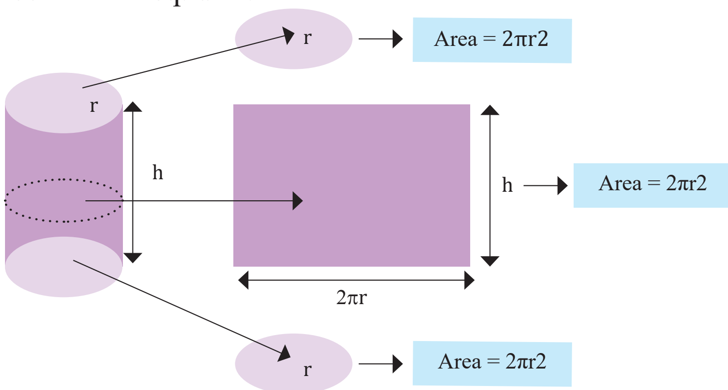
Individual task:

A company manufactures milk powder and wants to market it in right circular cylindrical tin pot.

The diameter of the tin pot is 16 cm and height is 24 cm. The company has decided to attach a packet surrounding the pot with 2 cm gap below and above of the pot. Determine the area of the packet.

Let's find formula to determine the whole surface area of a cylinder

We have learnt that the two ends of a right circular cylinder are identical circular area. If the radius of the circle is r unit, the area of the circle = πr^2 sq. unit. You have already learnt that if the radius of a cylinder is r unit and height is h unit, the area of the curved surface = $2\pi rh$ sq. unit.



So, we can tell from the figure above,

the whole area of a cylinder = Area of curved surface + 2 × Area of circle.

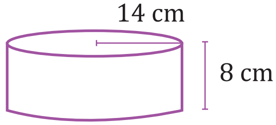
$= 2\pi rh + 2 \times \pi r^2$ sq. unit.

$= 2\pi rh + 2\pi r^2$ sq. unit.

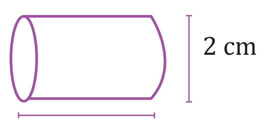
$= 2\pi r(h + r)$ sq. unit.

Individual task:

1. If there are two right circular cylinders in the following figures (i) and (ii), determine the whole surface area of them.



(i)



(ii)

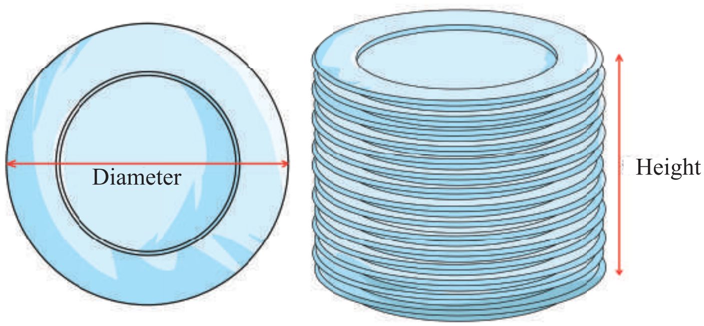
2. There are 24 round pillars in Namita's school. The diameter of each pillar is 30 centimetres and height is 4 metres. If it costs Tk.125 per square metre to paint, how much will it cost to paint all the pillars?

Volume of right circular cylinder

You have already learnt that volume of rectangular solid = Area of the base $\times$ height. Can you determine the volume of cylinder in the similar way?

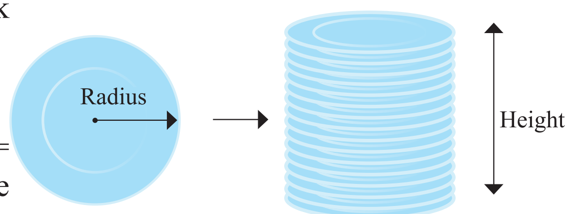
Let's try to understand observing some events.

1. Observe the following figure: You have certainly seen one dozen or more plates in a shop arranged in the following way. When many circular plates of same size are arranged one atop another, then the shape of the pile of plates is almost as cylindrical. Isn't it?



You will get the volume of the pile of plates determining the area of the lowest plate and multiplying it with the number of plates.

2. We can do the same cutting a thick circular paper.



We can tell from the above figure,

Volume of the pile of circular paper = Area of one circular paper × height of the pile.

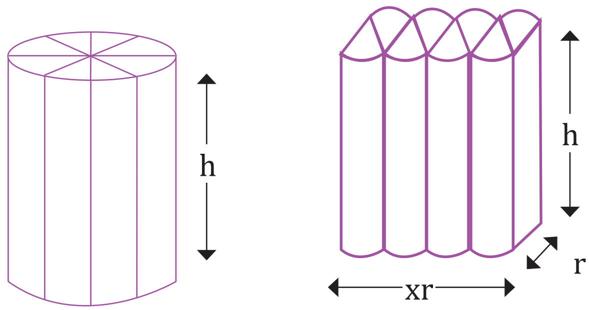
3. Let's make cylinder using plastic clay

Necessary materials: Plastic clay, knife and ruler or scale.

Process:

Step 1: Make a cylinder using Plastic clay whose height is h and radius of base is r.

Step 2: Cut the cylinder into eight parts with a sharp knife as shown in the figure below.



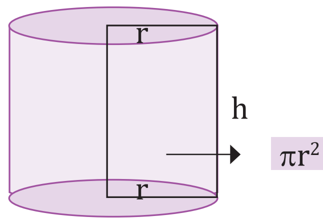
Step 1 and 2

Step 3

Step 3: Now make a hard structure like a rectangular solid as shown in the figure below where eight parts are attached to each other.

Observation and calculation:

Since, the rectangular solid is constructed connecting eight parts, then the length of the solid is πr unit, width is r unit and height is h unit. You have already learnt to determine the volume of a solid. Isn't it?



So, the volume of the solid = length × width × height

$= \pi r \times r \times h$ cubic unit

$= \pi r^2 h$ cubic unit.

That is, the volume of the cylinder = $\pi r^2 h$ cubic unit.

Since the base of the cylinder is a circular area and the area of the circle is πr^2 sq. unit,

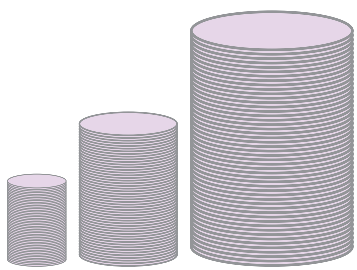
So volume of the cylinder = Area of circle × height

$= \pi r^2 \times h$ cubic unit.

$= \pi r^2 h$ cubic unit.

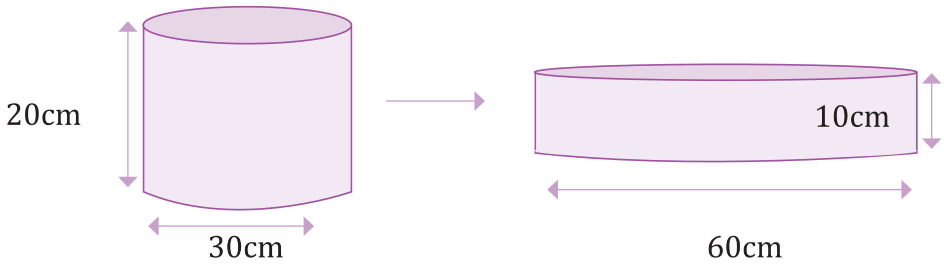
Individual task:

1. Look at the following picture. Here the dimensions of the cylinder are doubled in order. So what will be the effect on the volume? Give your opinion logically.



2. Look at the following picture. Here the second cylinder is constructed doubling the diameter and halving the height of the first cylinder. Find the

ratio of the volumes of the two cylinders.

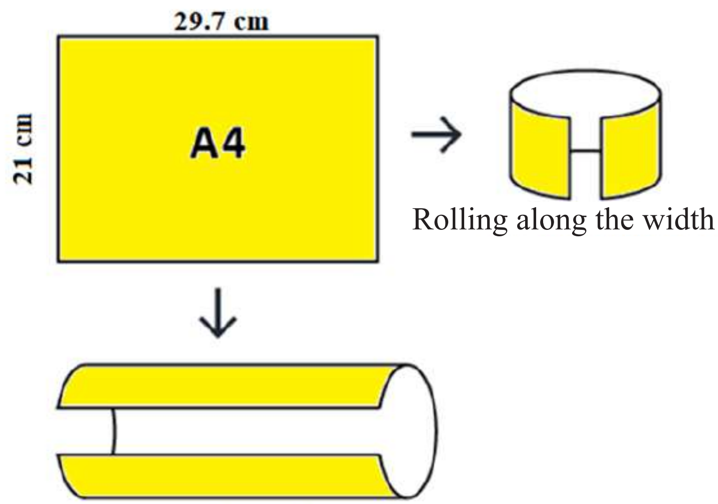


3. A biscuit company will make boxes of rectangular shape for biscuit packing. So , it plans to make two types of boxes.

a. length = 20 cm. width = 8 cm. height = 3 cm.
b. length = 12 cm. width = 10 cm. height = 4 cm.

Which type of box will be profitable for the company? Explain logically. The volume will remain the same as only changing the dimensions and the company will be benefited. Can you give such suggestions?

4. Make two cylinders rolling an A4 size paper along the length and width as in the figure below.



Rolling along the length

(a) Which cylinder made by you has a larger volume?

(b) What size of paper, if we cut from an A4 paper, will make both the cylinders equal? Justify your answer.

5. Cut two pieces of paper using scale measuring 21 cm. in length and 12 cm. in width. Now make two right circular cylinder by rolling one of the two pieces of paper along the length and the other along the width.

a. Determine the area of the curved surface and the volume?

b. Explain logically if there is any difference between the two cylinders.

6. A wooden box with a lid has outer dimensions of 10 cm. 9 cm. and 7 cm. The whole surface area of inner side of the box is 262 sq. cm. The thickness of the wood of the box is the same.

a. Determine the volume of the box.

b. Determine the thickness of the wall of the box.

7. The volume of a cylinder is 150 c.c. What are the possibilities of radius of base and height of the cylinder?

[Make a table and try assuming the value of radius and height.]

Algebraic Fractions: Addition-Subtraction, Multiplication-Division

Addition and Subtraction of Algebraic Fractions

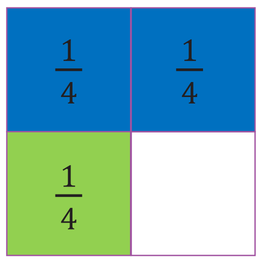
You have learnt about addition and subtraction of algebraic fractions in Class 6. You have learnt about arithmetical fractions as well. Let us now learn about the Addition and Subtraction of algebraic Fractions.

You must remember about arithmetical fractions; let us test if you can remember them.

First, take a piece of white paper and fold it into two equal folds. Think, what portion each fold is of the whole paper, and write in your exercise books.

Let us count. 1 piece of paper; 4 folds. So, each fold is $\frac{1}{4}$ portion of the whole paper. Since

The whole square shape is 1 paper. Hence,



The blue coloured part is = $\frac{2}{4}$ of 1 = $\frac{2}{4}$

The green coloured part is = $\frac{1}{4}$ of 1 = $\frac{1}{4}$

Total coloured part = $\frac{2}{4} + \frac{1}{4}$

Blue and green coloured part = $\frac{2+1}{4} = \frac{3}{4}$

Hence, white part = $(1 - \frac{3}{4}) = \frac{4}{4} - \frac{3}{4}$

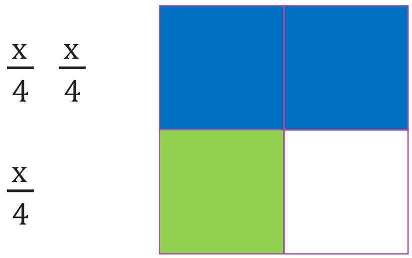
$= \frac{4-3}{4} = \frac{1}{4}$

Now let us look at that example from algebraic point of view. In this case, area of the paper is x unit^2. First fold the paper in two equal folds. Think what portion each fold is of the whole paper. Then fold each of the two portions into two equal folds ,
then into four parts. Like this, repeatedly divide into folds and write down the portions

in each of your exercise books. At the end, combine every two or three portions of the folded paper. Add and subtract the portions and compare with the real amount. Practice for the rest of the portions.

Now let's practice it with a paper whose area is x unit^2. Fold the paper in two parts, fold the two parts into four parts and use different colours to separate them.

If the area of the full square is x, then



The blue coloured part is $= \frac{2}{4}\ of\ x = \frac{2x}{4}$

The green coloured part is $= \frac{1}{4}\ of\ x = \frac{x}{4}$

Total coloured part $= \frac{2x}{4} + \frac{x}{4}$

Blue and green coloured part $= \frac{2x+x}{4} = \frac{3x}{4}$

Hence, white part $= (x - \frac{3x}{4}) = \frac{4x}{4} - \frac{3x}{4}$

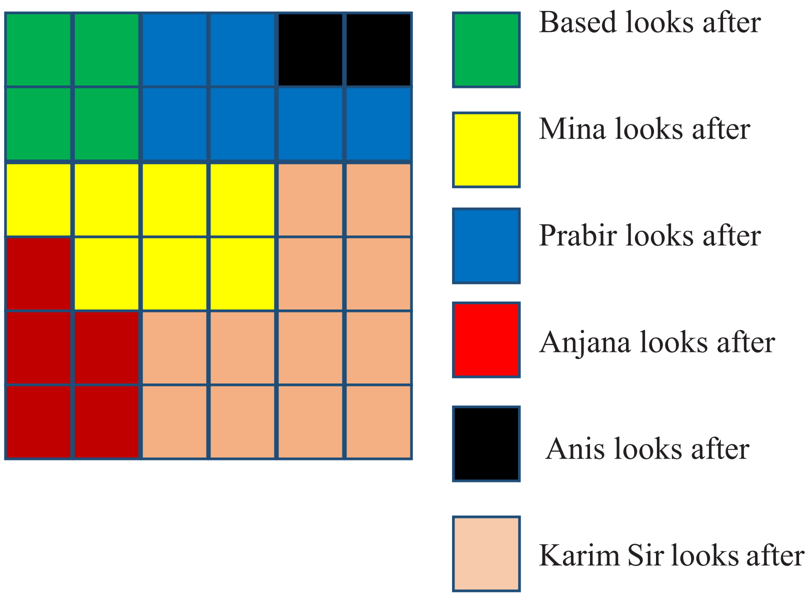
$$= \frac{4x-3x}{4} = \frac{x}{4}$$

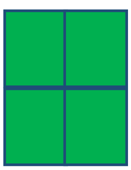
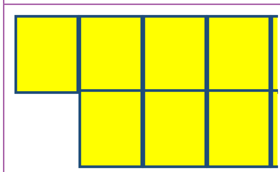
By now, you must have got your idea clear about Algebraic Fractions.

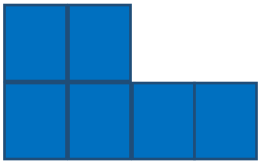
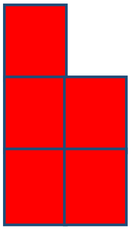
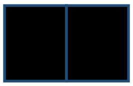
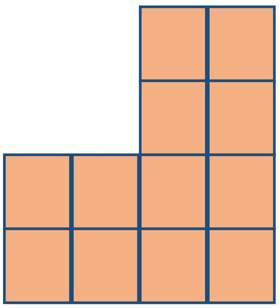
Worksheet 1:

Let's think about the map of a vegetable garden with different coloured parts. Total area of the garden is x. The students of Class 7 look after the garden. This year the responsibility of looking after the garden is given to Based, Mina, Prabir, Anjana and Anis respectively and Karim Sir, the teacher of Agricultural Science, oversees the remaining part. Now let us try to find what part of the garden is looked after by the students, and what part is overseen by Karim Sir, the teacher.

First take your exercise book and colouring pens. Now draw a square in your exercise book like the diagram below and colour the relevant areas. Then cut out the pieces with a pair of scissors and arrange them according to the colours.



	part looked after by Based = $\frac{4}{36}$ *of* x = $\frac{4x}{36}$
	part looked after by Mina = $\frac{7}{36}$ *of* x = $\frac{7x}{36}$

	Blue blocks part looked after by Prabir $= \frac{6}{36}\ of\ x = \frac{6x}{36}$
	part looked after by Anjana $= \frac{5}{36}\ of\ x = \frac{5x}{36}$
	part looked after by Anis $= \frac{2}{36}\ of\ x = \frac{2x}{36}$
	Total parts looked after by students $= \frac{4x}{36} + \frac{7x}{36} + \frac{6x}{36} + \frac{5x}{36} + \frac{2x}{36}$ $= \frac{4x+7x+6x+5x+2x}{36}$ $= \frac{24x}{36}$ Part overseen by Karim Sir = x - $\frac{24x}{36}$ $= \frac{12x}{36}$

Worksheet 2:

If Karim Sir gives away $\frac{1}{3}$ part of his responsibilities of looking after the garden to Based, then what will be the latest parts of the garden that will be looked after by Karim Sir and Based? Let us think about the matter.

At present, part overseen by Karim Sir $= \frac{12x}{36}$

Part after transferring some responsibility to Based $= \frac{1}{3}\ of\ \frac{12x}{36} = \frac{1}{3}\ of\ \frac{x}{3} = \frac{x}{9}$

Presently part overseen by Karim Sir $= \frac{x}{3} - \frac{x}{9} = \frac{3x}{9} - \frac{x}{9}$

$$= \frac{3x-x}{9} = \frac{2x}{9}$$

Presently part looked after by Based $= \frac{4x}{36} + \frac{x}{9} = \frac{4x}{36} + \frac{4x}{36}$ [expressing with common denominator, which is 36 here]

$= \frac{8x}{36}$ [numerator = sum of the transformed numerators]

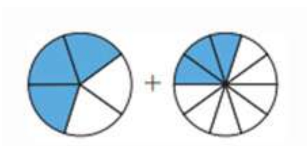
$= \frac{2x}{9}$

You must have noticed in the example above, that the fractions $\frac{4x}{36}$ *and* $\frac{x}{9}$ have different denominators. What should you do in such cases? In such cases, you should transform the two fractions with a common denominator.

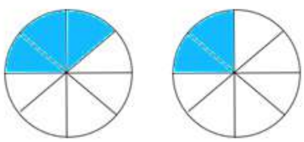
Individual Task:

(for questions 1 and 2, the area of the circle is x unit^2)

1. Determine the fractions from the model below and add them.



2. Subtract the second circle from the first circle



3. $\frac{1}{3}$ of a cane of length x is wrapped with red scotch tape, $\frac{1}{4}$ part wrapped with black scotch tape and the remaining part wrapped with white scotch tape. What is the part of the cane wrapped with white scotch tape?

4. Hena is a student of Class 7. She cultivated vegetables in $\frac{1}{3}$ part of the yard of her home, and flower garden in $\frac{1}{4}$ part. Determine in algebraic method, what part of her yard is empty.

Division of Algebraic Expression

Dividing a one term expression by a one term expression:

We have learnt from the idea of multiplication that, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

If we transfer the right hand side to left hand side and the left hand side to right hand side, then we can write $\frac{ac}{bd} = \frac{a}{b} \times \frac{c}{d}$.

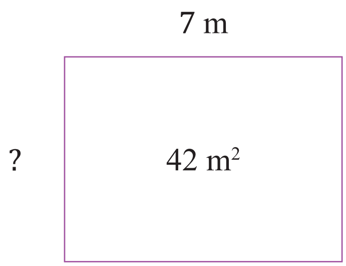
Now let us apply the above relation to algebraic expressions

$$\frac{-30x^6}{2x^4} = \frac{-30}{2} \cdot \frac{x^6}{x^4} = -15x^2$$

$$\frac{-21a^5b^4}{-3a^4b} = \frac{-21}{-3} \cdot \frac{a^5}{a^4} \cdot \frac{b^4}{b} = 7a^1b^3 = 7ab^3$$

$$\frac{12y^2z^2}{4y^2z} = \frac{12}{4} \cdot \frac{y^2}{y^2} \cdot \frac{z^2}{z} = 3y^0z^1 = 3 \cdot 1 \cdot z = 3z$$

If the area of a rectangle is 42 square metre and its length is 7 metre, then what is the width?



Try to represent it in a diagram. 7 metre

Here area = 42 m^2

Length = 7 m?

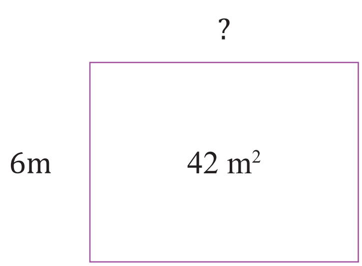
Width = $\frac{42}{7}$ = 6 m

Again, if the area of the rectangle is 42 square metre and width 6 metre, then what is its length?

Here, area = 42 m^2?

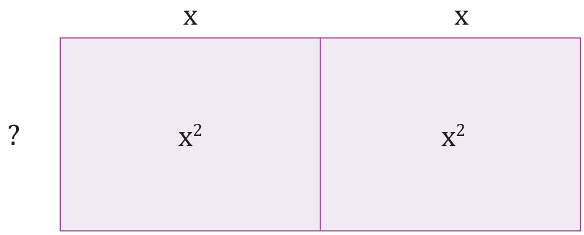
Width = 6 m

Length = $\frac{42}{6}\ m = 7$ m 6



Similarly, let us think about the area of the rectangle in terms of algebraic expressions.

The area of the floor of a room in a school is $2x^2$ square metres; if its length is $2x$, what is its width?



Area of floor of the room = $2x^2$ square metres

Length = $2x$ metres

Width = $\frac{2x^2}{2x}$ metre = x metre

$\boxed{a^m \div a_n = a^{m-n}}$

Example 1:

$x^5 \div x^2 = x^{5-2} = x^3$

$\boxed{a^m \div a^n = a^{m-n}}$

We know, $a \times (-b) = (-a) \times b = -ab$

hence-$ad \div a = -b$

Similarly $-ab \div b = -a$

$-ab \div (-a) = b$

$-ab \div (-b) = a$

$-ab \div (-b) = a$

$$-\frac{ab}{a} = \frac{a \times (-b)}{a} = -b$$

$$-\frac{ab}{b} = \frac{(-a) \times b}{b} = -a$$

$$-\frac{-ab}{-a} = \frac{(-a) \times b}{-a} = -b$$

$$-\frac{-ab}{-b} = \frac{a \times (-b)}{-b} = -a$$

Observe:

- Quotient of two expressions of same sign will be positive (+)
- Quotient of two expressions of opposite signs will be negative(-)

$$\frac{+1}{+1} = +1$$

$$\frac{-1}{-1} = +1$$

$$\frac{-1}{+1} = -1$$

$$\frac{+1}{-1} = -1$$

Example 2:

$24a^2bc^3 \div (-6abc^2)$

$$\frac{24a^2bc^3}{(-6abc^2)} = \left(-\frac{24}{6}\right) \times \frac{a^2bc^3}{abc^2}$$

$$= -4 \times (a^{2-1} \times b^{1-1} \times c^{3-2} = -4ac$$

Example 3:(-56xyz3) ÷ (-6x3y4z) = $\frac{-56xyz^3}{-6x^3y^4z} = \left(\frac{-56}{-6}\right) \times \frac{xyz^3}{x^3y^4z}$
$= \frac{28}{3} \times \frac{z^{3-1}}{x^{3-1}y^{4-1}} = \frac{28z^2}{3x^2y^3}$

Individual Task: Divide

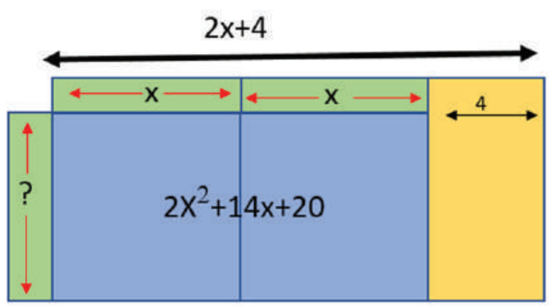
a. $\frac{24a^5}{-3a^2}$

b. $\frac{-18x^3y^2}{-6x^2y}$

c. $\frac{20a^3c^4d^2}{-5a^3c^3}$

A polynomial divided by a monomial expression

If the length of a field is increased by 4 metre and such that, the changed area of the field becomes $2x^2 + 14x + 20$, then what is its width?



Area of the rectangular field = $2x^2 + 14x + 20$ m²

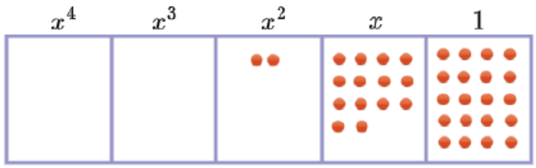
Length of the rectangular field = $(2x + 4)$ m

Hence, width of the rectangular field = $\frac{2x^2+14x+20}{2x+4}$

Let us use the method of game of buttons to divide the polynomial $(2x^2 + 14x + 20)$ by the monomial $(2x + 4)$ to find the width of the rectangular field.

Step 1: First draw some boxes, number of which should be more than the power of the polynomial to be divided. For example, here the highest power of x in the dividend is 2. So take the number of boxes as up to 3 or 4 powers.

Step 2: Select the 1st box from the right for the constant term, the 2nd box for the coefficient of term with x, 3rd box for the coefficient of term with x^2, 4th box for the coefficient term with x^3, 5th box for the coefficient of term with x^4 …….

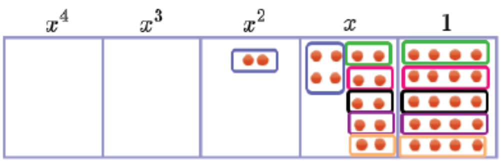


$2x^2 + 14x + 20$

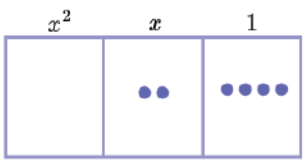
Step 3: Put buttons from the right, whose number is equal to the coefficients respectively, from the given problem.

For example, put 20 buttons in the 1st box from the right for the constant term, 14 buttons in the 2nd box for the coefficient of x, 2 buttons in the 3rd box for the coefficient of x^2.

Step 4: Make groups of buttons from the right consecutively, number of which is equal to numbers [or coefficients??] in the divisor.

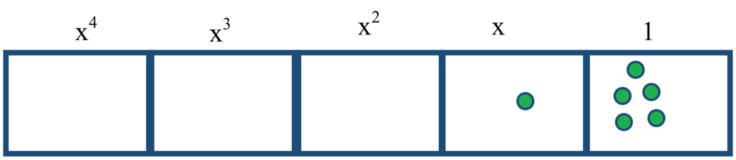


$2x^2 + 14x + 20$

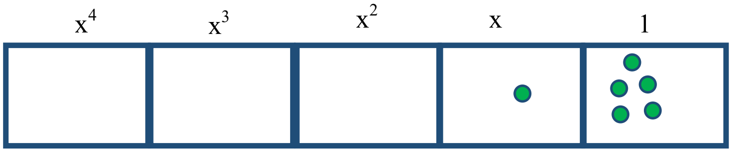


$2x + 4$

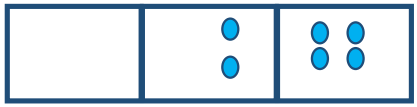
Step5: Put 1 button after one round, 2 buttons after 2 rounds, of different colours, and remove the previous groups [??]



Step 6: Considering the new coloured beads as the coefficients, match them with the variable. As a result, the following quotient will be obtained.



x+5



2x+4

Hence, $\frac{2x^2 + 14x + 20}{(2x+4)} = x + 5$

So, the width = (x +5) m

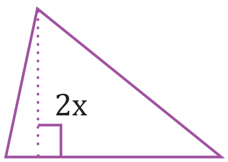
Example 4: Divide $4x^5 - 14x^4 + 6x^3 - 2x^2$ by $2x^2$.

Solution: $\frac{4x^5}{2x^2} - \frac{14x^4}{2x^2} + \frac{6x^3}{2x^2} - \frac{2x^2}{2x^2} = 2x^3 - 7x^2 + 3x - 1$

Example 5: Divide the first expression by the second expression: $3a^3b^2 - 2a^2b^3$, a^2b^2

Solution: $\frac{3a^3b^2 - 2a^2b^3}{a^2b^2} = \frac{a^2b^2\,(3a-2b)}{a^2b^2} = 3a - 2b$ $\therefore$ The quotient is $3a - 2b$

Example 6: The area of a triangle is $2x^2 + 3x$ square unit and if the height is 2x unit, what is the length of the base?



A=$2x^2$+3x

Solution:

$$\frac{4x^2 + 6x}{2x} = (4x^2 + 6x) \bullet \frac{1}{2x}$$

$$= 4x^2 \bullet \frac{1}{2x} + 6x \bullet \frac{1}{2x}$$

$$= \frac{4x^2}{2x} + \frac{6x}{2x}$$

$$= \frac{4}{2}\left(\frac{x^2}{x}\right) + \frac{6}{2}\left(\frac{x}{x}\right)$$

$$= 2x^{(2-1)} + 3x^{(1-1)}$$

$$= 2x + 3$$

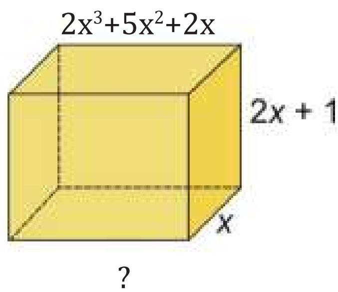
Task: Divide the 1st expression by the 2nd expression:

a) $3a^3b^2 - 2a^2b^3$, a^2b^2

b) $20x^3y + 10xy^2 - 15x^2y$, $5xy$

Dividing a Polynomial by a polynomial

Now let us try to measure the volume of a classroom. If the volume of room on the ground floor of a school is $2x^3 + 5x^2 + 2x$ cubic metre, height of the room = $(2x + 1)$ metre, and width = x metre, can you tell what may be the length of the room? Surely you remember the volume of a rectangular cuboid. Let us try to find it.



Volume of the room = $2x^3 + 5x^2 + 2x$ cubic metre

Height of the room = $(2x + 1)$ metre.

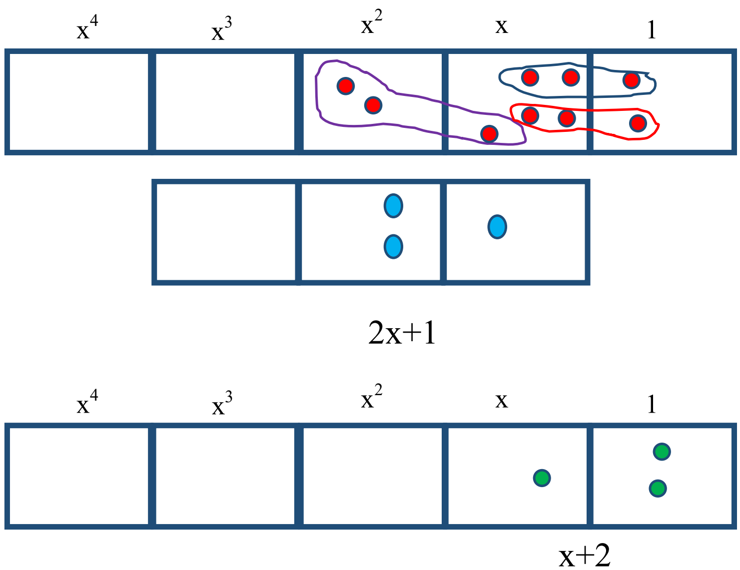
Width of the room = x metre

Length of the room = ?

Length of the room= $\dfrac{2x^3+5x^2+2x}{(2x+1)(x)} = \dfrac{2x^2+5x+2}{2x+1}$

Group task: The method of game of beads

The quotient = $(x + 2)$



Individual Task: Divide the polynomial (x^2 + 3x +2) by the polynomial (x +2), using the method of game of buttons.

Individual Task: Divide the 1st expression by the 2nd expression, using the method of game of buttons.

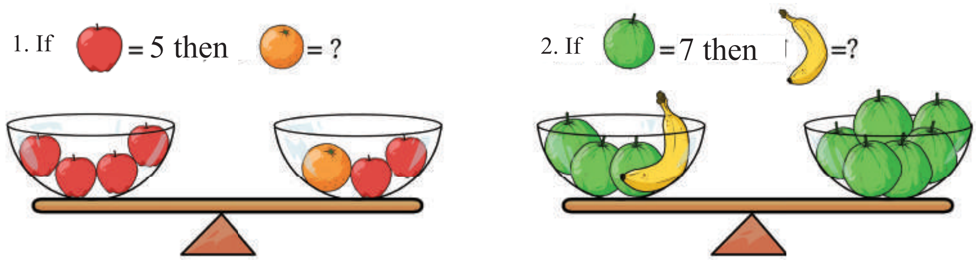
1. $24a^2b2c-15a^4b^4c^4-9a^2b^6c^2$, $-3ab^2$	5. $a^2+4axyz+4x^2y^2z^2$, $a+2xyz$
2. $a^3b^2+2a^2b^3$, $a+2b$	6. x^2-1, $x+1$
3. $6x^2+x-2$, $2x-1$	7. x^2-1, $x-1$
4. $6y^2+3x^2-11xy$, $3x-2y$	8. x^2+3x+2, $x+1$
	9. x^2-3x+2, $x-2$

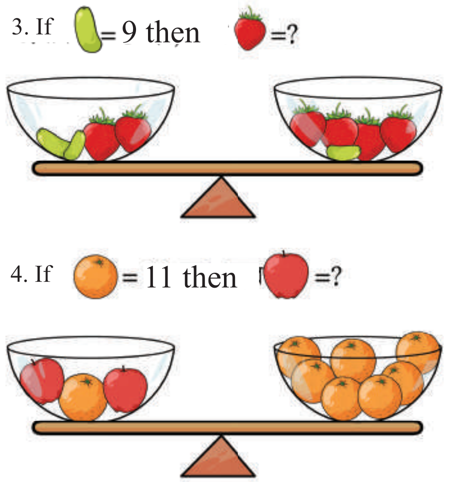
Linear Equation in One Variable

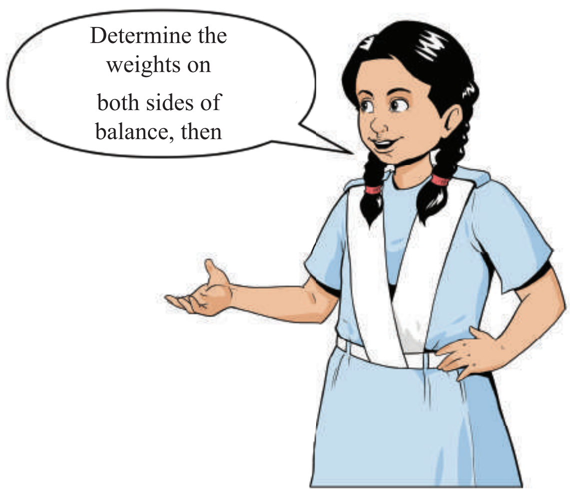
We have learnt about **equation** and **Linear equation**, in class 6 and learnt to construct equations from real life problems. We shall learn some techniques and applications to solve equations in this chapter of mathematics book for class 7.

In class 6, we learnt some rules to solve linear equations. Let us try to prove those rules realistically.

Determine what the unknown values may be from the equilibriums shown below and write down the answers in your exercise book.







Could you reach to any decision from the pictures detailed above? May we apply the decisions obtained to linear equations? Let us write the decisions in the exercise books.

Equations of equilibrium:

The rules of balancing scale may also be used to balance the equations.

The equilibrium of equations will be preserved if:

- Same amount is added on both sides
- Same amount is subtracted from both sides
- Both sides are multiplied by same quantity
- Both sides are divided by same quantity

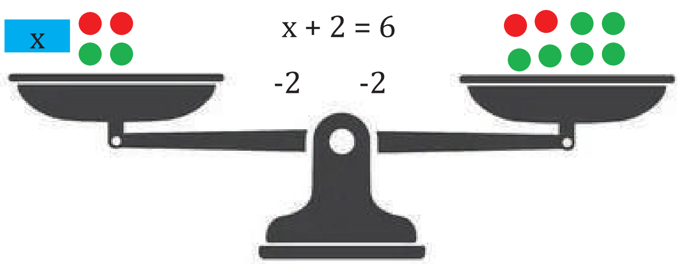
If the equilibrium of an equation remains valid, we shall not change the solution/ solutions of the equation.

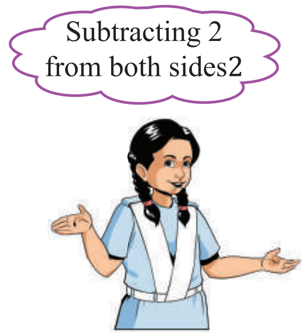
Let us try to apply the above decisions to equations. $x - 4 = 1$ The basic equation

Let us try to bring both sides to an equilibrium position, using the balance and the weights. Gradually put the weights (number of circles) on the balance. Bring both sides in equilibrium. At the end we shall get $x = 5$.

Decision1: If same number or quantity is added to both sides of an equation then both sides remain equal.

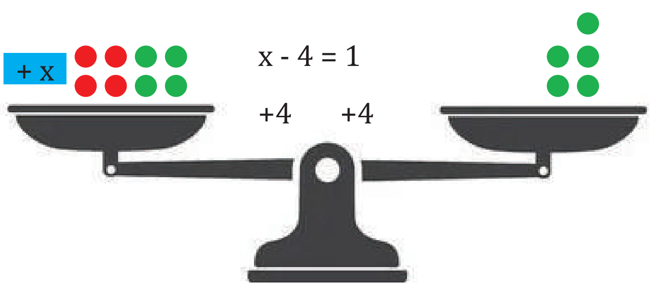
1. $x + 2 = 6$ The basic equation

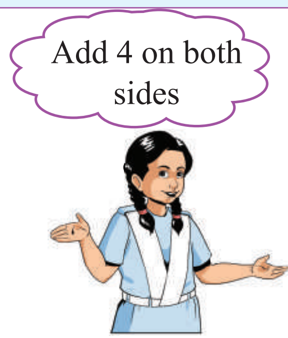




Let us try to bring both sides to an equilibrium using a balance and weights. Gradually add weights (number of circles) on the balance. Bring both sides to equilibrium. At the end we shall get $x = 4$.

Decision2: If same quantity or number is subtracted from both sides of an equation, both sides remain equal.





Task: Using a balance and weights, determine the changed equation of the equation $x+6=9$ and Determine the rules of multiplication and division.

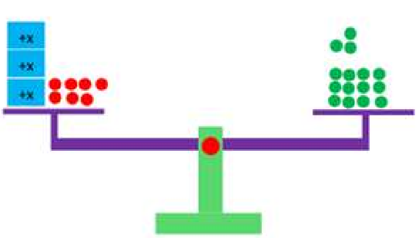
a) Add 3 to the equation b) Subtract 3 from the equation

c) Multiply by 4 d) Divide by 2

Rules of Equations:

1.

Observe the process of obtaining the equation $3x-7=15$ from the equation $3x=15+7$, using balance and weights.



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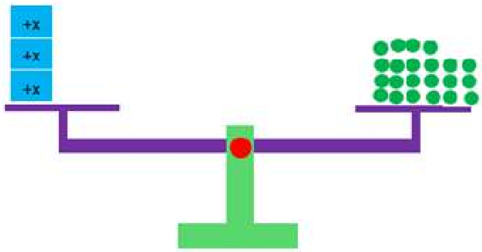


What have we learnt? We can call this process as the change of sides of the equation.

Now, using balance and weights, determine the changed equations of the following equations. Decide by observing the equations, where you can use exclusion principle of addition, exclusion principle of multiplication, cross multiplication, symmetry law.

Individual Task:

2. $7x = 20$ $from$ $7x + 5 = 25$

3. $3x + 2 = 2x + 1$ $from$ $5(3x + 2) = 5(2x + 1)$

4. $12x = 14$ $from$ $\frac{3x}{2} = \frac{7}{4}$

5. $7x - 4 = 5x + 2$ $from$ $5x + 2 = 7x - 4$

Now let us form linear equations and try to solve them.

Equation of Addition:

The height of Tajindong, the highest mountain peak of Bangladesh is 295 metre more than Keokaradong, the second highest mountain peak of Bangladesh. If the height of Tajindong is 1280 metre, then let us find the height of Keokaradong.

We shall form an equation to find the height of Keokaradong and solve it.

Let the height of Keokaradong = x metre

The height of Keokaradong + 295 = the height of Tajindong

Or, $x+295=1280$

Or, $x+295-295=1280-295$

Or, $x=985$

Both sides of an equation remain equal if same number is added to both sides.

Hence the height of Keokaradong = 985 metre

Equation of Subtraction:

There was a "safe hand wash programme" in Kabi Nazrul High School. There were 42 students absent in that programme and 915 students participated in the programme. Find the total number of students in Kabi Nazrul High School. We shall form an equation and solve it to find the total number of students in Kabi Nazrul High School.

Total number of students – number of students absent = number of students present

Or, $x-42=915$

Or, $x-42+42=915+42$

Or, $x=957$

Both sides of an equation remain same if same number is added on both sides. If $x = a$, then $x + b = a + b$

Hence, the total number of students in Kabi Nazrul High School is 957.

Equation of Multiplication:

Salam gets 300 taka per hour every day for his overtime work. He bought a mobile phone this month with 9000 taka received for overtime work. What is the total hour of his overtime work?

From the given information, we form an equation and solve it.

Overtime per hour × hours of overtime = cost of mobile phone

Or, $300 \times h = 9000$ h = number of overtime hours

Multiplying both sides of an equation by a non-zero number keeps both sides equal. If $x = a$ and $b \neq 0$, then $x/b = a/b$.

Or, $\frac{300}{300}$ h = $\frac{9000}{300}$

Hence, h = 30

Number of hours of overtime is 30 hours.

Equation of Division:

A shark can swim at an average speed of 20 miles per hour. What is the distance he can cover swimming in 24 hours at this speed?

From the given information, we form an equation and solve it.

Total distance covered ÷ total time = distance covered per hour

Or, $d \div 24 = 20$

Or, $d/24 = 20$

Or, $d/24 \times 24 = 20 \times 24$

Or, $d = 20 \times 24 = 480$

> Multiplying both sides of an equation by same number keeps both sides equal. If x = a, then xb = ab

Hence, total distance covered = 480 miles.

Explanation of the solution of linear equations:

Let us solve the following equation and check the equality of the equation.

$3\ (7 – 2x) = - 4x + 30$

Or, $21 - 6x = - 4x + 30$

Or, $-6x + 4x = 30 - 21$

Or, $-2x = 9$

Or, $2x = -9$

Or, $x= -9/2$

Hence the root/ solution of the equation is -9/2

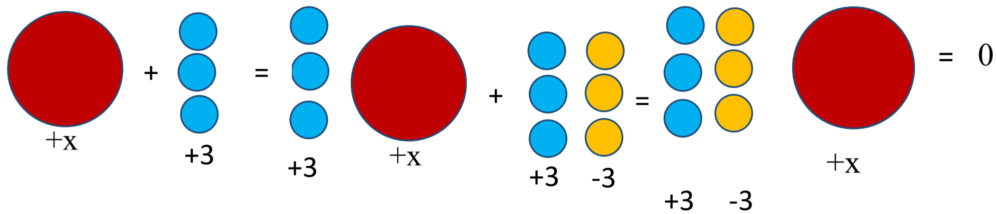
Check the equality:

Left Hand Side = $3(7\text{-}2x) =\{7\text{-}2\ (-9/2)\} = 3(7 + 9) = 48$

Right Hand Side = -4x + 30 = -4(-9/2) + 30 = 18 + 30 = 48

Substituting the value of the root/solution in both sides, we obtained that the values of Left Hand Side and Right Hand Side are equal.

Example: Solve the linear equation x+3=3, by cutting out papers and colouring them.



Individual task

Using the equilibrium of balances, solve the following equations.

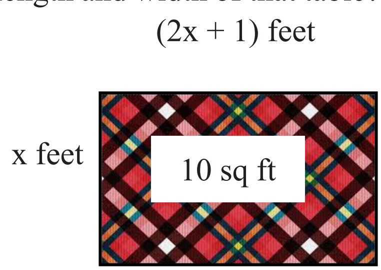
1. What is the number when adding 5 to the double of it will give the sum 25?
2. Sum of two numbers is 55 and 5 times the bigger number is equal to 6 times the smaller number. Find the two numbers.
3. Gita, Rita and Mita together have 180 taka. Gita has 6 taka less than Rita and Mita has 12 taka more. Determine who has what amount?

Quadratic Equations in Single Variable

We have learnt different types of polynomials. We have learnt forming linear equations in single variable and finding their solutions, using different types of polynomials. Now using quadratic polynomials, we shall learn to **form Quadratic Equations in Single Variable** and get familiar with the usage of them.

Worksheet 1

Suppose you have decided to spread a tablecloth on your study table. A cloth of area 10 square feet, whose length is 1 foot more than twice the width, will be spread. What do you think we should do if we want to determine the length and width of that table? Can anyone tell what will be the width? Since the width is unknown, let us assume that the unknown quantity of width is x foot. Now can you tell what will be the length of the unknown area? Since the length is 1 foot more than the twice the width, then how shall we express the length in terms of unknown quantity? The length must be (2x + 1) feet. Let us represent these information in a picture.



Can you tell what the equation formed with these information will be like?

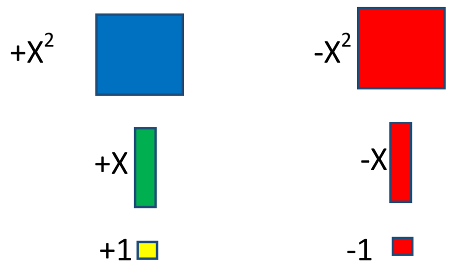
Width of tablecloth is x feet and its length = (2x +1) feet. Hence the area is x(2x + 1) sq. ft. The equation formed will be x(2x + 1) = 10. Hence, $2x^2 + x - 10 = 0$.

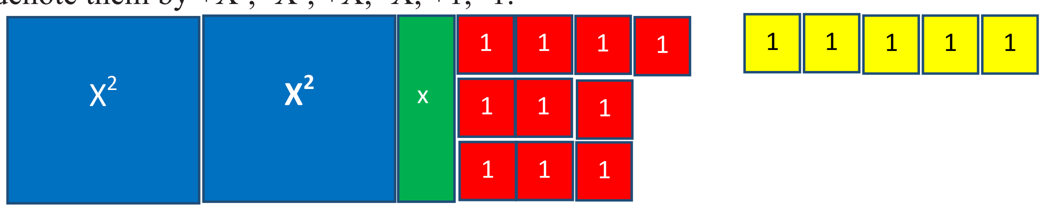
Have you noticed the equation? Can you tell how many variables are there in the equation $2x^2 + x - 10 = 0$? You will certainly say that there is 1 variable. Now the question is what is the highest power of the variable? You will certainly say it is 2. Can you tell what is the coefficient of x^2? What is the coefficient of x? And what is the constant?

Now let us denote the coefficient of x^2 by a, coefficient of x by b and the constant term by c. So we get $ax^2 + bx + c = 0$, which is known as the quadratic equation in English. Do you know what the form of this kind of equation is called? This is an ideal form of equation, where a, b, c are real numbers and $a \neq 0$.

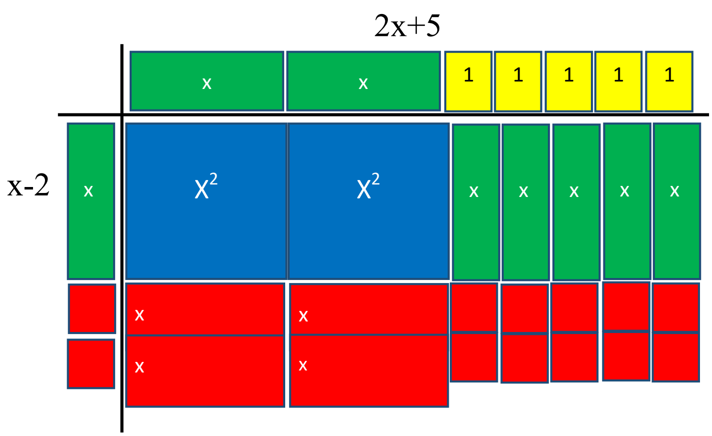
Now let us display the quadratic equation $2x^2+x-10=0$ by the method of cutting papers as follows.

First take some red-, green-, blue- and yellow-coloured papers to solve the equation. Cut the papers accurately as the following shapes and denote them by $+X^2$, $-X^2$, $+X$, $-X$, $+1$, -1.





Now represent the equation with pieces of papers.



Using the pieces of papers give different shapes to the equation to form rectangles or squares.

Area of the rectangle $(2x+5)(x-2)=0$

Hence $(2x+5)(x-2)=0$

Or, $(2x+5)=0$ Or $(x-2)=0$

$\therefore x=-5/2$ or $x=2$

N.B. $x = -5/2$ is not acceptable, as the length of cloth cannot be negative (–).

Individual Task: Write in the ideal form $ax^2 + bx + c = 0$ and find the values of *a. b, c*

	Ideal form	a, b, c
$3x-2x^2=7$	$2x^2-3x+7=0$	2, -3, 7
$(x-7)(x+7)=3x$	....	....
$5+2z^2=6z$		
$2x(x-3)=15$		
$5w(7w-2)=10w+1$		
$4y-3y(y)=9$		
$A+2a^2-19=5a^2$		

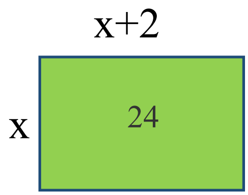
Worksheet: 2

Forming Equation:

Picture of a rectangular field is given below. Let us form an equation from the information given and check if that is a quadratic equation or not.

In the picture, length of rectangular field = (x + 2) metre, width = x metre

Area = (length × width) m^2



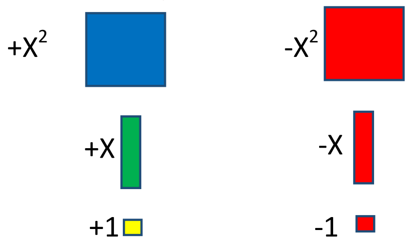
$24 = (x + 2) \text{ x } m^2$

$x^2 + 2x - 24 = 0$

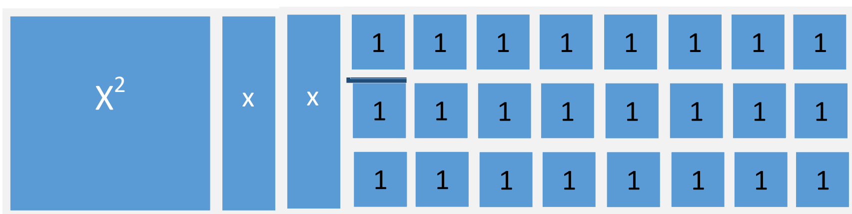
Hence $x^2 + 2x - 24 = 0$ is a quadratic equation.

Now let us display the above equation $x^2 + 2x - 24 = 0$ using the method of cutting papers.

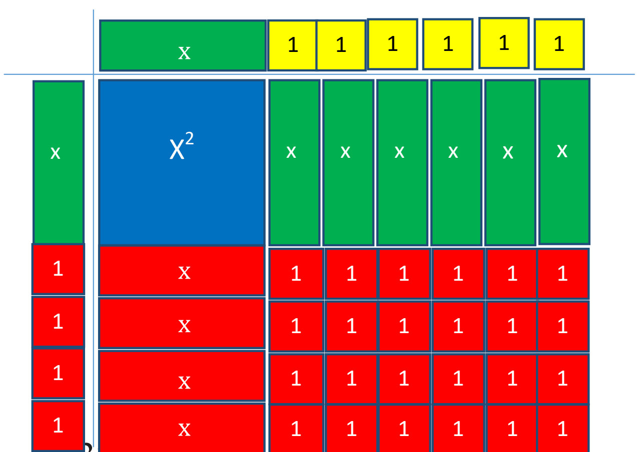
First take some red, green, blue and yellow colour papers. Cut the papers accurately as the following shapes and denote them by $+X^2$, $-X^2$, $+X$, $-X$, $+1$, -1.



Now present the equation using the pieces of papers.



Using the pieces of papers give different shapes to the equation to form rectangles or squares.



Area of the rectangle, $(x + 6)(x - 4) = 0$

Hence, $(x + 6)(x - 4) = 0$

Or $(x + 6) = 0$ or $(x - 4) = 0$

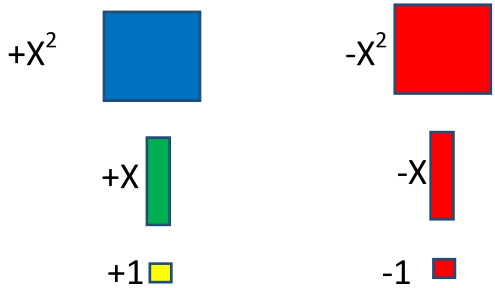
$x = -6$ $\quad\quad$ $x = 4$

N.B. $x =- 6$

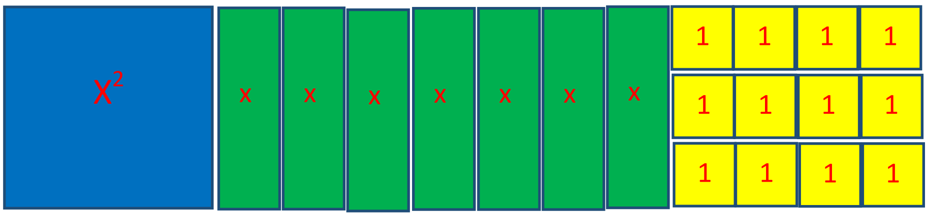
Example: Solve the quadratic

equation $x^2 + 7x + 12 = 0$, using paper cutting.

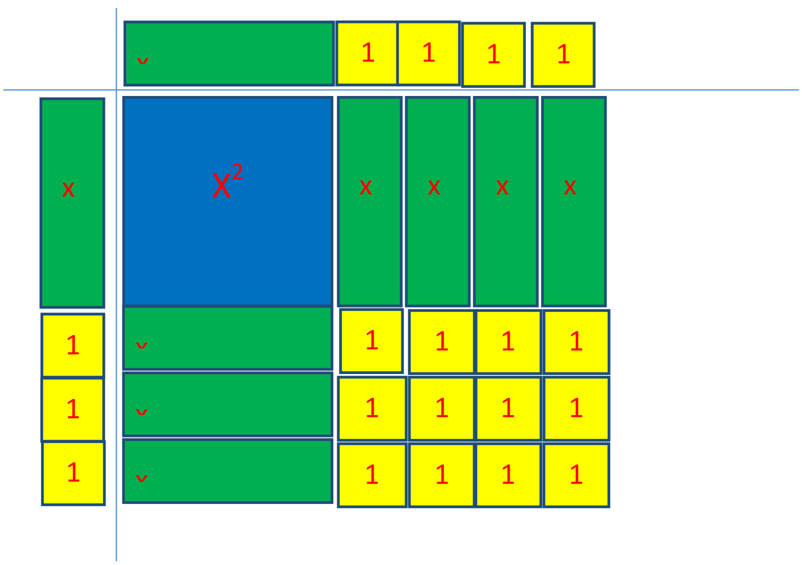
First take some red, green, blue and yellow colour papers. Cut the papers accurately as the following shapes and denote them by $+X^2$, $-X^2$, $+X$, $-X$, $+1$, -1.



Now present the equation using the pieces of papers.



Using the pieces of papers give different shapes to the equation to form rectangles or squares.



Area of the rectangle, $(x + 4)(x + 3) = 0$

Hence, $(x + 4)(x + 3) = 0$

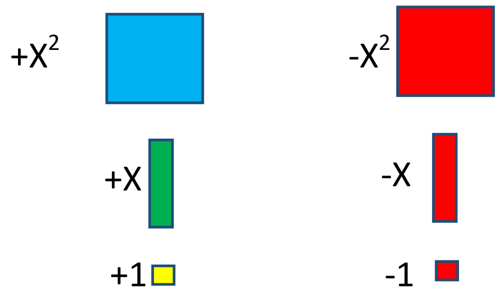
Or $(x + 4) = 0$ or $(x + 3) = 0$

$x = -4$ or $x = -3$

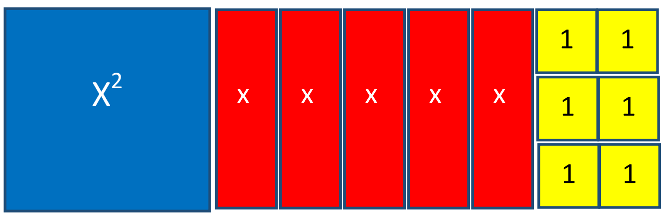
Task: The length of a pond is 4 metre more than its width and its area is 105 sq. M. Form an equation using the information given.

Example: Solve the quadratic equation $x^2-5x+6=0$, using paper cutting.

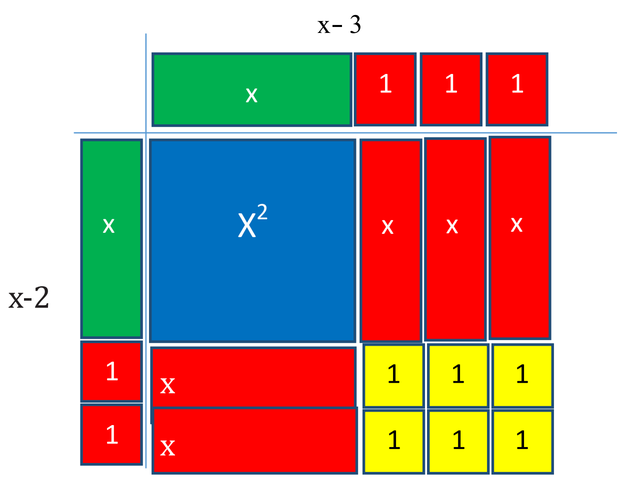
First take some red, green, blue and yellow colour papers. Cut the papers accurately as the following shapes and denote them by $+X^2$, $-X^2$, $+X$, $-X$, $+1$, -1.



Now present the equation using the pieces of papers.



Using the pieces of papers give different shapes to the equation to form rectangles or squares.



Area of the rectangle, $(x - 3)(x - 2) = 0$

Hence, (x - 3) (x - 2) = 0

Or   (x - 3) = 0 or (x - 2) = 0

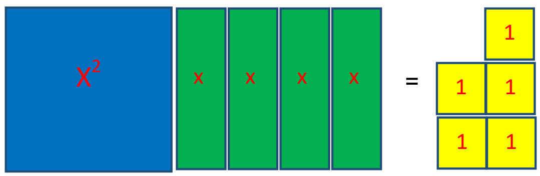
x = 3   x = 2

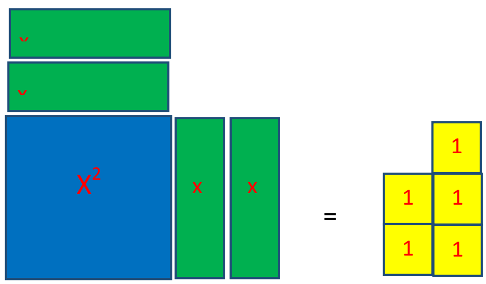
Example: Solve the quadratic equation $x^2+4x=5$ using paper cutting.

First take some red, green, blue and yellow colour papers. Cut the papers accurately as the following shapes and denote them by $+X^2$, $-X^2$, $+X$, $-X$, +1, -1.

Now present the equation using the pieces of papers.

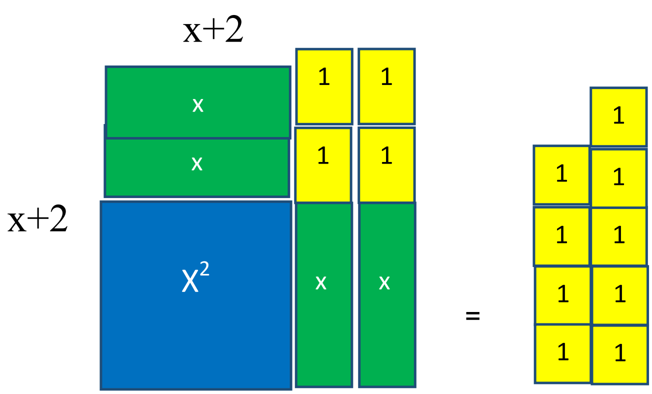
$x^2+4x \quad = \quad 5$





$x^2+4x+4 \quad = \quad 5+4$

Using the pieces of papers give different shapes to the equation to form rectangles or squares.



$(x + 2)^2 = 5 + 4$

$\sqrt{(x + 2)^2} = \sqrt{9}$

$x + 2 = \pm 3$

$x = 3 - 2$ or $x = -3 - 2$

Solutions: $x = 1$ or $x = -5$

Task: Cutting papers, solve the quadratic equation $x^2 + 6x - 7 = 0$

Individual Work

Form quadratic equations and solve them by cutting papers:

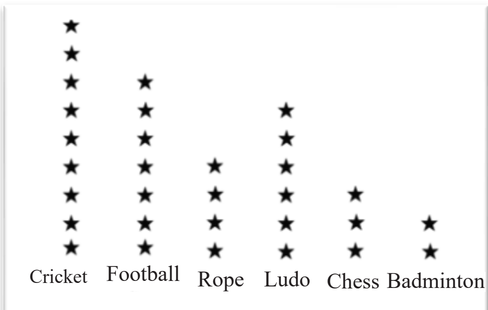
1. The sum of the digits of a two-digit number is 15 and their product is 56. What is the number?
2. The area of the floor of a rectangular room is 192 sq. m. The area remains unchanged if the length of the floor is reduced by 4 metres and the width is increased by 4 metres. Determine the length and width of the floor.
3. The length of the hypotenuse of a right-angled triangle is 15 cm and the difference of the lengths of the other two sides is 3 cm. Determine the lengths of those two sides.
4. The length of the base of a triangle is 6 cm more than twice its height. If the area of the triangular field is 810 sq. cm, then what is its height?
5. The amount of subscription collected from each student of a class was 420 taka when each paid the amount equal to the number of students in the class. What is the number of the students in the class, and how much did each pay?
6. Each student of a class paid subscription equal to the amount of 30 paisa more than the number of students in a class and the total collected money was 70 taka. What is the number of students in that class?

Information Exploration and Analysis

The Headmistress of Mitu's school declared in the Assembly today that sports equipment will be bought for all the classes. Everyone made a cheering sound! But then Madam said, "Don't cheer already! There are different types of sports equipment and you did not tell me which ones to buy. Or will you accept whatever I buy?" Everyone said, "No madam, no! We want the opportunity to choose". Madam said "Okay, if there is a problem in making decisions, then you will solve it".

It was Mathematics class of Rafique sir in the First period. He entered the classroom saying, "We should complete the task of sports equipment for your class, what do you say?" Everyone was excited, "Yes Sir". Sir said, "Then tell me what equipments can we buy?" Raihan said, "Sir, Cricket bat and ball!" Shumi Said, "Sir, Skipping Rope!" Nayan said, "We need a Football, Sir!" Zinat and Bithi said, "We want a Ludo set, Sir!" Mahim and Kaushik said, "Then it will be good to have a Chess set, Sir!"

Out of excitement, nobody noticed that Sir was writing whatever the boys and girls were saying on the board. Now Sir said, "Do you want anything else?" Mitu said nervously, "Sir, we could play Badminton in school if we got a set." Sir smiled and wrote Mitu's Badminton on the board too. Then he said, "I have a rough idea of what you want. But I must make a final information list of the equipment needed for Class Seven Joba , to give to the Headmistress Madam, understood?"



Picture 7.1 on p 1

Saying this, sir said, "Okay, let me see, how many will raise their hands for the Cricket Set?" Ten students raised their hands. Sir drew ten stars above the column of Cricket. Then he said, "Football?". Students raised their hands. In this way, stars were drawn above the name of each game, which looks something like the following picture.

Sir said, "Wow, very nice! See, how all of us together made a table! Now tell me, what do you understand?" Mishu is the scientist of the class. He said, "Sir, Everyone in our class like Cricket most." Sir said, "That's brilliant, Mishu! Can you all say, how did Mishu know this?" Mitu said sadly, "Sir, most stars are in the column of cricket. Everyone raised their hands for Cricket. And the least number of stars are in the column of Badminton, only two. Nobody, except me and someone else wants to play Badminton." Sir said, "You are right! Then why are you sad?" Robin said from the side, "Sir, Mitu thought Badminton set will not be bought because of having the least votes." Sir laughed loudly, "No, no! The Headmistress Madam said, if a single person wants, even then that equipment will be bought. Do not worry about it Mitu!" Then Mishu stood up and said, "Sir, then I want a magnifying glass in the class. Then I can see if there is a leak in the football. Sir said, "Okay! Remember this while buying the set of equipment for your science classes. Today, let us find only the sports and games equipment". Everyone laughed.

"Now listen. When I wanted to know from you at the very beginning, what games you wanted to play, and what you had told me, those are **Information**. Why is it necessary to collect such information? Suppose I asked you verbally, you would answer whatever you wanted. In that case a rough idea would have been obtained. That means there would have been some gap for your demands."

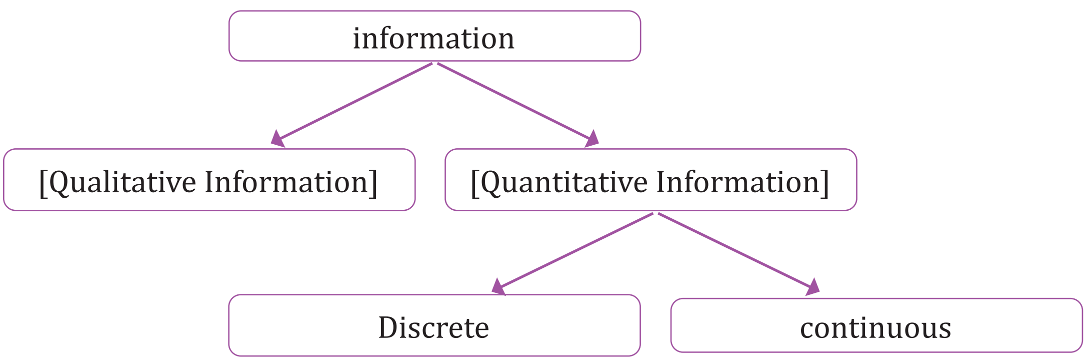
"It is not enough just to collect. We do not eat the vegetables straight away after getting them from the market, they need to be cooked, they need to be processed- finally they become edible. Just like that, we need to process the collected information. Hence, we arranged the information in a table that is we arranged them. Then the information collected a while ago, became meaningful Data in front of us. We understood what our need was."

Processed Data become meaningful information in front of us. We can take a decision by looking at this Data. Do you want to do this work with the demand for sports equipment in your class? If we prepare a table of data with demands and opinions from all and present it to the head of the institution surely s/he will not say no, isn't it?

Now can you tell after thinking a while, what type of information have we obtained about the games equipments? Is it quantitative or qualitative? Okay, let me help a bit more – the quantitative information can be expressed with mathematical numbers. And qualitative means descriptive, there is nothing numerical in it, you cannot measure its amount. Now think and do the following individual task:

What type of information do you think are there in the Table 7.1? put a tick sign (√) on the left side of the table below. Write down a reason for your decision on the empty space on the right.

Tick symbol	Type of Information	reason
	qualitative	
	quantitative	



Do check the correct answers and reasons from your teacher. Up until now we have learnt that information is mainly of two types: **qualitative** and **quantitative**. Quantitative information is again of two types. We can see what they are, from the following chart:

It is necessary to get clear idea about discrete and continuous quantitative information to perform the following tasks. Both are numerical information, but there are some differences between them.

Let's talk about discrete information first. The name suggests this type of information is in units, not connected or falls in a serial. The main characteristics of discrete information are that, they are not changeable with time, several measuring numbers are also not combined.(??) These numerical data may be whole numbers or fractions, or both. For example, our National Martyrs' Monument is 150 feet high, and the height of Eiffel Tower in Paris is exactly 300 metres. Again, the length of our Padma Bridge is 6.15 km. In this case, note that the heights of Martyrs' Monument and Eiffel Tower or the length of the Padma Bridge is constant. As many times as we measure them, the numerical values of these will not change. There is more to it – your present shoe size, the total number of students in your class this year, the total number of stairs in your school etc. Can you write three more examples of discrete information in the following table?

Discrete Information
1.
2.
3.

Now we talk about continuous information. The main characteristic of the continuous information is that its value is not fixed. This information can have any value. For example, the temperature of a day. Think about the temperatures at the time when you came to school and at noon- are they same? Also, will it be same at the end of the day or in the evening? Are not the temperatures after a heavy rain at noon and in sweet sunshine in the afternoon different? So, if you want to talk about the temperature of a day, you have to combine some information together. There are more examples, the height of a tree in one year, the production of paddy during last five years in Bangladesh, etc. Can you give three examples of continuous information in the following table?

Continuous Information
1.
2.
3.

Individual task

Now let us complete an individual task. This task will clear all the ideas about different types of information. This time also, you must fill up a table with different data. If you process the data, they will be transformed into information. Here we shall assume that the data is already processed. The classified data in the following table will be cited as information. You can collect the information from your home and school. Fill in the blank cells with your collected information and put a tick sign (√) for the correct type of information.

Ser No	Description	Information	Types of Information		
			Qualitative	Quantitative	
				Discrete	Continuous
1	Your name				
2	Your age				
3	Which class are you in				
4	Establishment year of your school				
5	Your height				
6	Number of members in your family				
7	Electricity bill of last month at your house				
8	Number of books in your room				
9	How many kg of rice bought last month				

Individual task

You must have enjoyed the task of collecting information. However, it is not over yet. You need to know how the information was collected. Some methods of collecting information are given in the table below, put tick sign(√) on the empty spaces for the methods you used. Write in the empty spaces if you have used any other method.

Serial	Method of Collecting Information	Used
1	by observation	
2	by tests	
3	from some file or database	
4	from internet	
5	by questioning people	
6	from news papers	
7		
8		

9		

There are other methods to collect information besides the ones shown above. For example, by interviewing, using a questionnaire, by discussing with specific groups, etc. It is not easy to collect information. Many a time, you need to use several methods, several sources. Sometimes it takes a lot of time and is expensive to collect Data or information.

You must have acquired a primary idea about collecting information from the individual work. Our next task is a group project. The subject of the project, necessary directions and helping ideas are given below.

Group task

1) Divide the whole class into 5 groups with the help of the teacher. Each group will work on a project. The projects are:

a. Favourite colours of the classmates small pictures for each project
b. Favourite food of the classmates
c. Heights of classmates
d. Number of students absent in class daily, for a month
e. Number of family members of the classmates

2) Write down in the empty space, which project you are participating in:

3) Write down the number of members in your project, in the empty space:

4) Plan to collect Data:

Serial number	Subject	Plan
1	Source of Data	
2	Reason for selecting source	
3	Medium of collecting Data	
4	Logic for choosing medium	
5	Type of Data	
6	Logic for choosing the type of Data	
7	Date of collecting Data	
8	Date of classification of Data	
9	Date of processing Data	
10	Type of presentation of processed Data	

11	Reason for the choice of type of presentation	
12	Date of submission of the final report	

Now get to know the tasks of your group. Complete the following steps according to the plan above.

1. Prepare the materials for collecting information
2. Collect information
3. Prepare a Frequency Distribution Table for the collected data using one of the two applicable methods:
    a. In counting few discrete or qualitative data, use Tally marks directly, to find the frequency
    b. For many continuous data i) determine the range ii) determine the class interval; iii) determine the class number iv) determine the frequency using tally marks.
4. Draw a Frequency Distribution Graph, using one of the three applicable methods from the following:
    a. Line-graph
    b. Histogram
    c. Pie chart
5. Prepare a report. You can include the following answers into your report:
    a. What is the purpose of your project?
    b. What information-data are necessary for the project?
    c. In which method and from which source did you collect the information-data? Why did you think that this source was the best?
    d. How did you process the data? Show each step of the processing.
    e. Show the graph of the processed data. What is the reason of choosing the type of graph you chose?
    f. Write down, what you have decided from the processed information.

At this stage, you can start collecting data and information with your group. You have to process the information and data after collection. You have learnt in Class Six that these data and information are part of statistical work. Hence the processing should also be in the statistical methods. You must have some basic idea and should know about methods, before processing the information and data in statistical methods. These are explained in the remaining part of this chapter. In this case, your teacher will help you and give you necessary advice. Remember, to go ahead with the work of your project, there is no harm in looking at these ideas again and again. But you must do some individual tasks to get these ideas cleared.

Presentation of Data

You have already known that numerical information is statistical data. These data are usually unarranged and necessary decisions cannot be taken from unarranged data. Hence these data must be arranged or entered in a table. You have learnt in Class Six how to arrange the unarranged data in order of their values. In this Chapter, we shall try to learn how to arrange the unarranged data through classification and enter into a table.

What is the meaning of the classification of something? Suppose there are 40 students in your class. If all of you enter the classroom together and sit anywhere you want, does it not look haphazard? The classroom remains unarranged. But if a bench or a seat is allotted to each student, then everyone can sit in an orderly manner in a disciplined way. If each person sits on the allocated seat, the classroom looks arranged. It is similar for the data. If there are many unarranged data, you cannot find a meaning out of it. But depending on the number of data, taking suitable intervals, dividing the data into some classes, it remains arranged, and you can analyse it easily. This arrangement is called classification. Let us find the answers to the following two questions.

- In which process can you classify data?
- Has there been any classification of data at any stage in this Chapter?

Frequency Distribution Table

The most dependable scientific method of arranging unarranged data through classification is to prepare the Frequency Distribution Table. There is no fixed rule to prepare a frequency distribution table. The explorers or the researchers have made the frequency distribution table, in different ways, in different times, according to their own needs. Let us see with an easy example, how this table can be prepared.

Suppose your teacher wants to know what the students of your class like to eat for breakfast. There are so many things to eat for breakfast in the morning, but in this case, items will be limited to only rice, bread, tea and biscuit. The teacher gave the responsibility of collecting the information to Mahir, the class captain. Mahir first went to Sharif to ask, Sharif chose rice; Mahir put a mark with a pencil in the block of rice. Mitu chose tea and biscuit; he put another mark on that block. In this way, he put a mark for each one of the 40 students in the class, according to their choices. He collected all the data as in the table below and submitted to his teacher.

<table>
<tr><th>Food of choice</th><th>Tally marks</th></tr>
<tr><td>Rice</td><td>||||||||||||||||</td></tr>
<tr><td>Bread</td><td>|||||||||||||||||</td></tr>
</table>

Tea-biscuit	

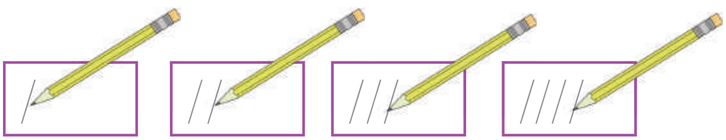
We call each mark a tally. Notice the table, each tally denotes the data for each student. In other words, each tally denotes a number for the counting. So each tally is a frequency number.

After knowing what frequency is, Priti added another column on the piece of paper written by Mahir, to see how many data was obtained against each of rice, bread and tea-biscuit. The name of new column was given frequency or number of students.

Food of choice	Tally mark	Number of students
Rice		16
Bread		17
Tea-biscuit		7
	Total	40

Now the teacher said, the use of tally for counting by Mahir is a correct method for counting frequency. But there is an easier way to count using tally.

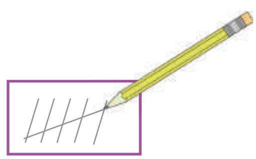
We can use the tally marks for 1 – 4 as follows:



1 2 3 4

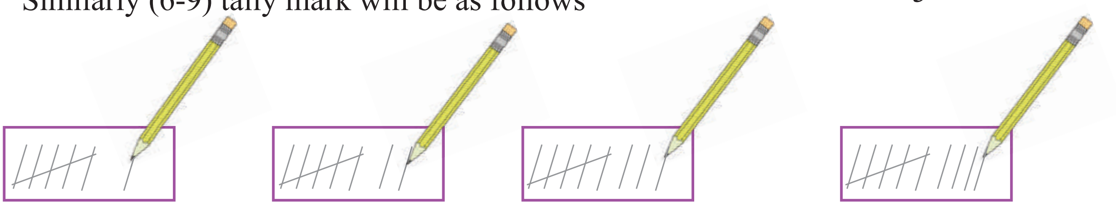
But the 5th tally mark is to be given across, as follows:

So, when counting tally, you see something like consider it as 5.



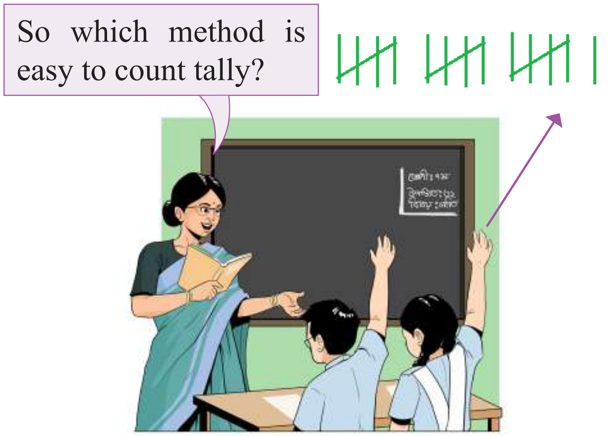
5

Similarly (6-9) tally mark will be as follows



6 7 8 9

Now quickly show the frequency 29 using Tally in the empty space below.

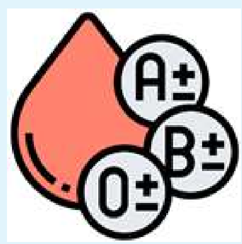


Hence, we can present the table prepared by Mahir and Priti as follows:

Food of choice	Tally marks	Frequency or Number of students
Rice		16
Bread		17
Tea-biscuit		7
	Total	40

It can be seen from the table that, in that class, maximum 17 students like bread for breakfast in the morning and minimum 7 students like tea-biscuit.

Individual Task: Collect blood group for all the students in your class. Then answer the following:



a. Present the information by preparing a frequency distribution table.
b. What is the blood group that most students belong to?
c. What is the blood group that least students belong to?

Up to this point, we have worked with a simple frequency distribution table. Number of data in this table is few, so need less time for management of this. Besides, the information in this table is discrete. Remember what discrete information is? However, frequency distribution table for continuous information is a bit different. Let us first learn how we can prepare a frequency distribution table for continuous data.

If the number of information or data is large and continuous, then presenting the data by the rule above is difficult and time consuming.

Suppose wages per hour (in Taka) for some 60 labourers are given below:

50, 40, 58, 45, 55, 48, 52, 60, 42, 55, 45, 62, 61, 57, 58, 61, 42,43, 50, 44, 37, 57, 43, 62, 53, 43, 42, 45, 51, 54, 62, 38, 37, 49, 55, 64, 55, 60, 61, 40, 38, 34, 41, 36, 38,

51, 38, 62, 45, 47, 52, 39, 51, 33, 49, 63, 64, 65, 5o, 55

If you want to prepare a frequency distribution table for wage of each labourer, then the table will be very large, and it will take long time to prepare it. Again, there are possibilities of making mistakes too. In this case you can arrange the unarranged data very easily through classification and it will be easier for you to present it using frequency distribution table.

To prepare the frequency distribution table for continuous data, usually the following steps are followed.

1. Finding range → 2. Finding Class Interval → 3. Finding Class Number → 4. Finding frequency

1. **Finding Range:** The difference between the highest and lowest values is called the Range. So, the formula for finding the range = (highest value – lowest value) + 1

The highest number of the data is 65 and the lowest number is 33.

Hence the Range of the data = (65 – 33) +1 = 33

2. **Finding Class Interval:** It is necessary to find the Class Interval after finding the range of explored data. For that, the data is divided into some classes, with convenient differences. Usually, this division is done depending on the amount of data. There is no fixed rule for division into classes. But there is a maximum and a minimum value for each class. For any class, the minimum value is called the lower limit, and the maximum value is the upper limit. The difference of the upper limit and lower limit of any class is the class interval for that class. For example, 1 – 10, 11 – 20, 21 – 30 etc, each is a class. Here, for the class 1 – 10, the lower limit is 1 and the upper limit is 10. Class Interval is (10 – 1) +1 = 10. It is better to keep the class interval always same.

3. **Finding Class Number:** The Class Number is the number of classes the range is divided into.

That is Class Number = $\frac{Range}{Class\ Interval}$ (rounded to a whole number)

The Range of wages per hour (in Taka) of labourers = 33, suppose Class Interval = 5

Then Class Number = $\frac{33}{5} = 6.6 \approx 7$ (rounded to next whole number). [to represent approximate value, $\approx$ symbol is used.]

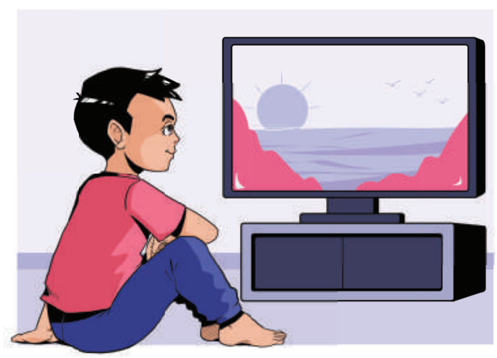
Now answer a complex question. Why is the Class Number 6.6 rounded to the next whole number 7? Instead, do you think there would be any problem in preparing the table, if the previous whole number 6 was taken?

The numerical value of the information data will be in one of the classes. It is necessary to put tally marks for each numerical value against the class and the frequency is counted through this. The number of tally marks in a class will be the frequency of that class, which will be written in the frequency column.

Now let us prepare the frequency distribution table for those wages of the 60 labourers. There are two done for you.

Class Interval	Tally marks	Frequency
30 - 35	$\mid\mid$	2
36 - 40	$\cancel{\mid\mid\mid\mid}\ \mid\mid\mid$	8
	Total	

Individual task: Collect the information from your classmates, how many hours each watched television last week. Then through classification of the unarranged data, prepare the frequency distribution table and show it to your subject teacher.



Determination of Actual Class Limits:

To perform the following group task, it is essential to learn to determine the class limits and actual class limits. Instead of telling which is what, let us understand it through the group task.

Group Task: Determine the weights (in Kilogram) of each student of the class, by dividing the class into few groups. Then prepare a list by writing down the weight obtained beside the name of the relevant student.

Fill up the empty spaces below with the frequency or number of students, by the classified data obtained by weighing (in Kg) all the students of your class.

Class Interval or Weight (in Kg)	Frequency orNumber of students
31 – 35	
36 – 40	
41 – 45	
46 – 50	
51 – 55	
56 – 60	
61 – 65	
66 – 70	
Total	

Now if two new students, weighing 45.5 Kg and 50.5 Kg get admitted to your class, then in which class interv al will you include them? You cannot create a new Class interval to include them. Again, you cannot include them in the class intervals 41 – 45 or 46 – 50 either. Since the difference between upper limit and lower limit of two consecutive class intervals is 1, hence the two data 45.5 and 50.5 cannot be included in any class intervals. In such cases, divide the difference of 1 into two equal parts (1 ÷ 2 = 0.5) and add to the upper limit and subtract from the lower limit of each class interval to determine the actual class intervals.

For example, consider the two class intervals 41 – 45 and 46 – 50.

The upper limit of 41 – 45 = 45 and the lower limit of 46 – 50 = 46.

Hence the difference of the upper limit and lower limit is (46 – 45) = 1.

Hence half of the difference is (1 ÷ 2) = 0.5.

So, the actual class interval for 41 - 45 will be = (41 - 0.5) - (45 + 0.5) which is 40.5 – 45.5

Similarly, the actual class interval for 46 – 50 will be = (46 – 0.5) – (50 + 0.5) which is 45.5 – 50.5

In this case actual class intervals for weights of the students of your class will be as follows:

Class interval or Weight (Kg)	Actual Class interval
31 – 35	30.5 – 35.5
36 – 40	35.5 – 40.5
41 – 45	40.5 – 45.5
46 – 50	45.5 – 50.5
51 – 55	50.5 – 55.5
56 – 60	55.5 – 60.5
61 – 65	60.5 – 65.5
66 – 70	65.5 – 70.5

Now it is possible for you to include the weights of the new students in the table. But can you see any other problems for including their weights? Note that there is 45.5 in both the class intervals 40.5 – 45.5 and 45.5 – 50.5

According to your opinion, which class interval should you use for 45.5? If you include 45.5 to both the classes 45.5 will be counted twice. Hence according to the rule, 45.5 should be included in the Class 45.5 – 50.5, not in the Class 40.5 – 45.5. Now think and tell in which class should 50.5 be included?

Hence the weights 45.5 and 50.5 should be included in the classes 45.5 – 50.5 and 50.5 – 55.5 respectively.

Then the new frequency distribution table will be-

Class Interval (In kgs)	Actual Class Interval	Class Number or Students
31-35	30.5-35.5	
36-40	35.5-40.5	
41-45	40.5-45.5	
46-50	45.5-50,5	
51-55	50.5-55.5	
56-60	55.5-60.5	
61-65	60.5-65,5	
66-70	65.5-70.5	
	Total	

Individual Task:

Observe the frequency distribution table and answer the following questions.

This is a frequency distribution table of daily wages of 650 labourers of a factory

Class Interval (Daily wages in Tk)	Frequency (Number of labours)
500 – 600	45
600 – 700	50
700 – 800	90
800 – 900	150
900 – 1000	200
1000 – 1100	50
1100 – 1200	35
1200 – 1300	20
1300 – 1400	5
Total	650

a. What is the Class interval?
b. Which Class has the highest frequency?
c. Which Class has the lowest frequency?
d. What is the upper limit of the Class 900 – 1000
e. Which two Classes have same frequency?

You have learnt plenty of tasks by this time for processing and analysing data with the help of your teacher. At this stage, collect blood pressure of 20 people from amongst your neighbours. Then write down the answers to the following questions and submit to your subject teacher on the next day.

a. Among the two types of data collected, which are discrete, and which are continuous? Explain giving reasons.
b. For which type of data, do you need the actual Class limits to prepare a Frequency distribution table and why?
c. Find out the ranges for the two types of data.
d. Taking appropriate Class Intervals find the Class Number of the data.
e. Taking appropriate Class Intervals, present the number of family members of each of your classmates in a Frequency Distribution Table.
f. Prepare a Frequency Distribution Table for the blood pressure of your neighbours, taking an appropriate actual Class Interval.

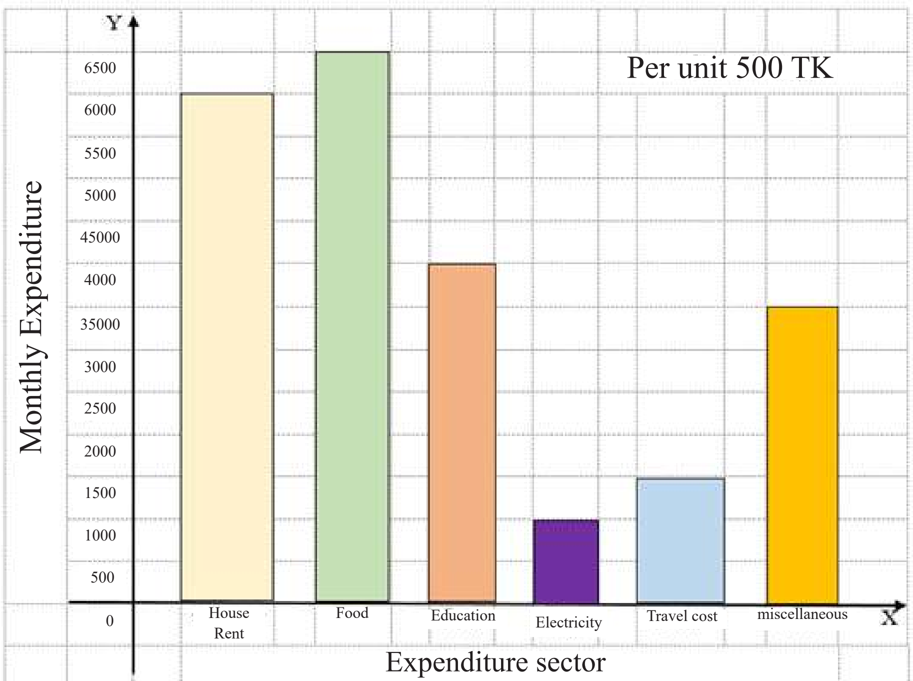
Graphical Representation of Data:

We have already discussed representing information through a Table. Let us consider representing information in another method. That is, by pictures or by graphical representation of information or data. There is a saying, **one picture is equivalent to a thousand words.** The idea which cannot be expressed in a thousand words, many a time that idea can be expressed in a picture. Besides graphical representation of information and Data is a very old acceptable method.

Bar Graph/ Bar Diagram

Every year, Mridul goes on a tour with his parents. To manage the expenses for the tour, of course they save money throughout the year. Mridul's father writes down the monthly income and expenditures in a notebook, covered with red paper. The interesting matter is that, at the end of each month he sits down with everyone and draws a picture. During the drawing, he says, by looking at this picture you can tell in advance which item is incurring more expenses and how much you need to reduce. Besides you can decide about the expenses of the next month, and you have an advance idea of how much savings is possible. The picture in the notebook of expenditures of Mridul's family is given below.

Looking at the picture, Mridul remembers, he has seen pictures of this type in previous



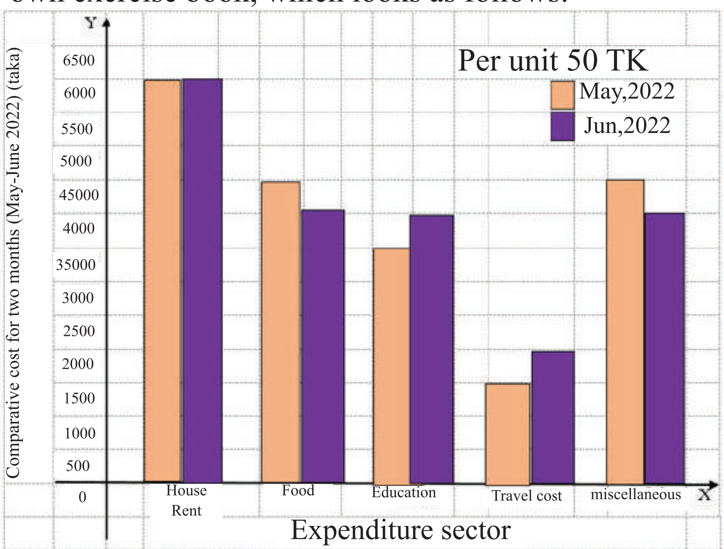
class, the name of which is Bar Graph or Mridul observes that the information and data of the monthly expenses of his family has been represented in the picture drawn by his father.

All of you observe the picture drawn by Mridul's father carefully and write down the answer to the following questions in your exercise book.

a. What is the name of the diagram?
b. What type of information and data will be available from the diagram?
c. What is the unit in the vertical direction of the diagram?
d. Which item cost the most in the month of concern?
e. Which item cost the least in the month of concern?
f. What was the expenditure on education in that month?
g. What are the advantages of representing the information and data in bar graph?

Individual Task: Collect the information and Data of the monthly expenditure of your family for any one month. Then represent the item wise family expenses by a bar graph and submit it to your subject teacher in the next class, for evaluation.

Mridul found that there is another diagram in the notebook. Mridul drew a similar diagram in his own exercise book, which looks as follows:



After drawing the picture, Mridul observed that for each item there are two bars drawn side by side. So, this type of bar graph may be named Compound Bar Graph. He also noticed that in the diagram, the information and Data of item wise expenditures of two months have been presented side by side. The interesting matter is that, from the diagram the difference between the item wise expenses of two months can be determined easily. Mridul decided mentally that he will find out the reasons of the differences of the item wise family expenses from his father.

If someone wants to see the comparative expenses for three or four months in one diagram, then the information and data can be represented by drawing three or four bars side by side.

Individual Task: Collect the information and data of the item wise expenditures of the family for three continuous months. Then draw the compound bar graph to represent the information and answer the following questions:

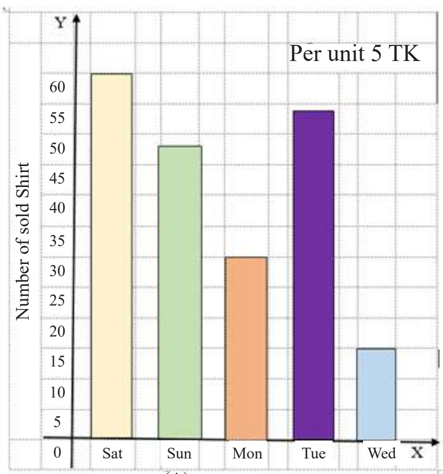
a. What information and Data did you obtain from the Bar Graph?

b. Explain the reasons for the differences in the expenses of different items.

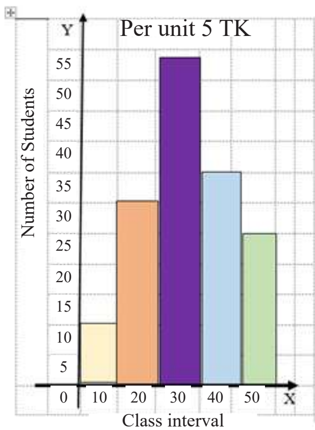
c. "The compound bar graph has a special role in preparing a uniform family budget" – explain with your opinion.

Histogram

Observe the two diagrams below carefully:



(A)



(B)

Work in Pairs: Find out the similarities and differences among the two diagrams above. After discussing with your classmate write them down in the fixed place of your textbook. Then, either of you present your observation in the classroom. The other student of the pair will write the feedback you obtain from other classmates in the textbook or exercise book.

The similarities among the two diagrams (a) and (b):

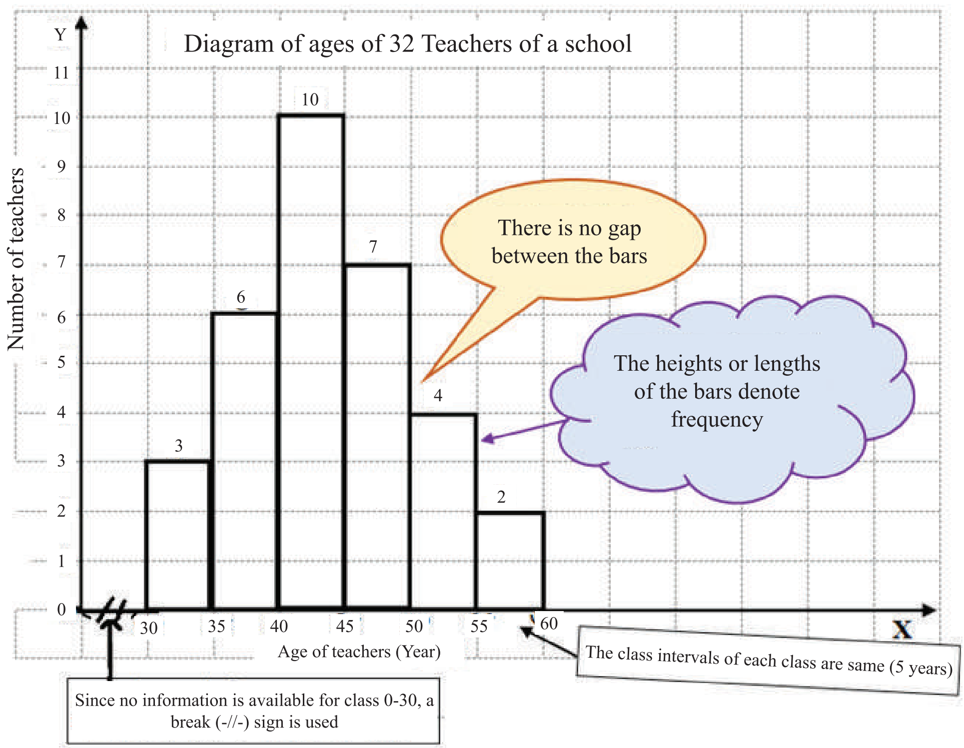
Diagram (a)	Diagram (b)

The differences among the two diagrams (a) and (b):

Diagram (a)	Diagram (b)

After your observations and discussions, we can say, in diagram (b), the bars are drawn side by side. That is, there is no gap between the bars. The actual class intervals are taken along the horizontal or x – axis and the number of students or the frequency is along the vertical or y - axis. The width or base of each bar or rectangle is the class interval and the height or the length is the frequency. If information or data is represented by such diagrams, then that is called Histogram. The word histogram was first used by the English mathematician Carl Pearson.

The area of each rectangle in the Histogram is proportional to the frequency of the respective rectangle. Again, since the widths of all the rectangles are equal, the heights of the rectangles are proportional to the frequency of the respective rectangles. For this reason, we only draw the heights. Observe the following histogram:



Observe the above Histogram and answer the questions below:

a. How many of the teachers are more than 50 years but less than 55 years old?
b. How many teachers are less than 45 years old?

Individual task:

a. Collect information about the different ages (in years) of the families of your neighbours and fill up the table.

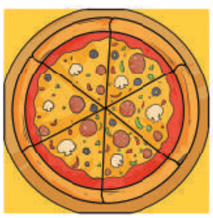
Age (years)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Number of people									

b. Draw the histogram from the prepared table.

c. Which class interval accommodates the highest number of people? Determine from the histogram.

Pie Chart or Circle Graph

Another method of expressing the data in pictures is Pie Chart or Pie Diagram or Circle Graph. Many of you probably know what a pie is. You can say that pie is a kind of foreign *pitha*. These are circular and thick. You can see in the following picture what a pie looks like.



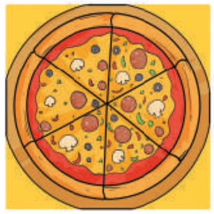
Are you tempted to eat? Pie is indeed a very tasty food. Observe, how it is sliced. There are few other round-shaped foods like pie known to us. We eat *chitoi pitha* and *bhapa pitha*. Both the *pithas* are circular. You must also know about pizza, an item from Italy, is also circular. You must be thinking, why we are talking so much about circular food. Before answering that, look at the pictures of food items below. Then there is a small task.

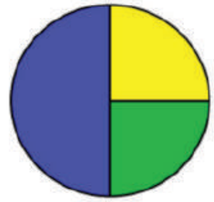
Chitio Pitha BhapaPitha Pizza

Now suppose you and your friend Ratul are sharing a circular food, it may be a *chitoipitha*, or a pizza or a pie. Look at the first picture below. Write down what portion you have got and what portion Ratul has got. After a while, another friend of yours, Shumi joined you. Look at the second picture, and write down in the empty space, who obtained what portion.

Ratul's Portion:...................... %

Your Portion: .... %



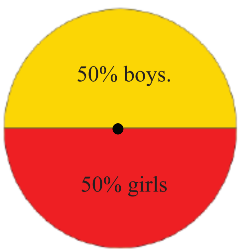


You must have understood in the task above that, percentage has been explained by dividing a circle. It is possible to divide a circle in many portions, just like a pie or pizza can be divided into equal or unequal portions in some fixed methods. The method of dividing a circle in triangular shapes and expressing them in percentages is called Pie Graph or Pie Chart.

We have already learnt some of the styles of presenting information and data. You also know that the annual document containing the plans for incomes and expenses of a country is called its national budget. This budget expresses data-information of the income sources of government and the codes of expenses through various diagrams. The diagram below presents one such proposal:

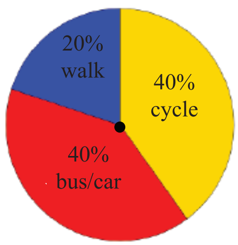
The above diagram is also a kind of presentation of information and data. It presents the code-wise annual expense of the government through a circle. It is also a graph. It is called Pie-chart or circle graph. It got such a name as it looks like the food called pie.

a.



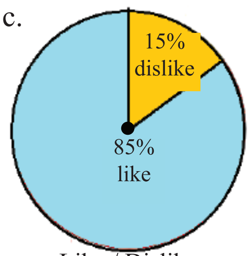
number of boys and girls in your class

b



use of transport to school

c.



Like / Dislike Mathematics

Some pie charts are given below, in which there are different information of students of a class. Let us try and see whether it is possible to explain or not.

Individual Task: Write 5-10 sentences in the frame below describing what you learnt about the students of that class from the pie charts above.

Now let us see, how to prepare a pie chart. We know that the sum of the angles at the centre of a circle is 360^0. And the angle at the centre for each piece of the circle will be a fraction of 360^0. Any Statistics represented as a fraction of 360^0 will be a pie chart.

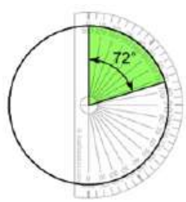
Abraham collected information about the choices of fruits of the 250 students of class seven, which is shown in the following table:

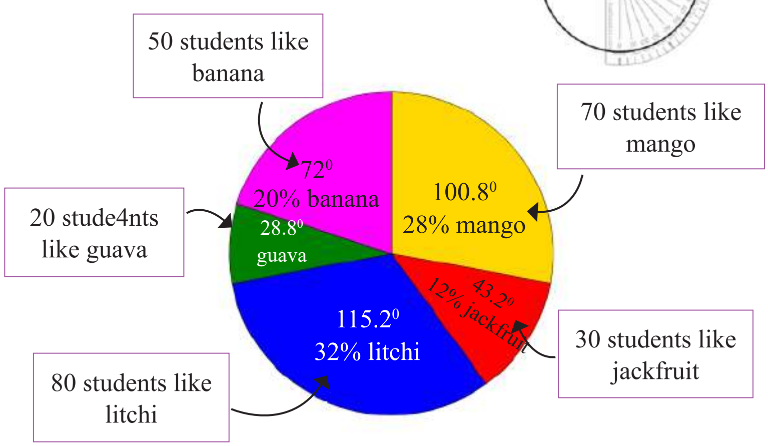
Fruits of choice	Mango	Jackfruit	Lichi	Guava	Banana	Total
Number of Students	70	30	80	20	50	250

Let us prepare a Table of the data collected by Abraham to show them in a pie chart.

Fruits of choice	Number of students	Express in percentage	Angle at centre for each piece of circle (in degrees)
Mango	70	$\frac{70}{250} \times 100 = 28\%$	$\frac{70}{250} \times 360 = 100.8°$
Jackfruit	30	$\frac{30}{250} \times 100 = 12\%$	$\frac{30}{250} \times 360 = 43.2°$
Lichi	80	$\frac{80}{250} \times 100 = 32\%$	$\frac{80}{250} \times 360 = 111.2°$
Guava	20	$\frac{20}{250} \times 100 = 8\%$	$\frac{20}{250} \times 360 = 28.8°$
Banana	50	$\frac{50}{250} \times 100 = 20\%$	$\frac{70}{250} \times 360 = 72°$
Total	250	100%	360^0

Let us create each part of the circle measuring each angle at the centre with a compass.



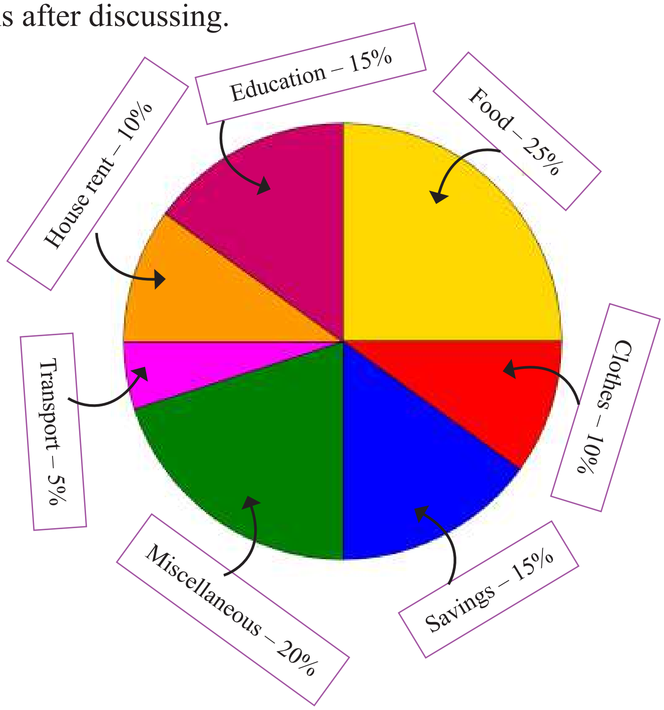


Now we'll draw a pie chart according to the table above and represent the information and data through that.

Individual task: **K**now the ages (in years) of all the members in your family. Prepare a Table with the data of ages of everyone. Then use the table to draw pie chart and present it

Work in Pairs:

In the picture below, expenses of different items in the family of Shuman Chakma for one month is shown along with the savings. Observe the picture well and answer the following questions after discussing.



a. Shumon Chakma saves Tk 3000. What is the total expenditure of Shumon Chakma in that month except the saving?
b. What is the expenditure of Education?
c. What is the highest expenditure of Shumon Chakma and how much, in Taka?
d. Find the angle in the centre of the Pie Chart for each item.

Individual Task:

1. Collect 10 pieces of information from your daily life. Classify the information through a tree.
2. Look around your home or residence; there are different types of trees. Do you know the names of all the trees? If necessary, take help from your guardians.

Now check, how many trees are there of each type. If you want, you can draw the pictures of the trees too. You can even write down the approximate heights of the trees in units of your choice. Use tally marks to fill up the following table with the number of different types of trees and the total number of trees.

Name of Trees	Tally marks	Approximate height	number
....			

Answer the following questions:

a. Which trees have you seen the most?
b. Which trees have you seen the least?
c. How many trees are there in total?
d. According to your observation, which tree has the maximum height and what is it?
e. According to your observation, which tree has the minimum height and what is it?
f. Using the names of trees and the number of trees obtained from the table, draw a bar graph.
g. Determine the range of the heights of the trees.
h. Taking appropriate class interval, determine the class number of the heights.
i. Prepare and fill up a table like the one below in your exercise book and draw a histogram according to the table.

Heights of trees or class interval (according to your unit)	Actual Class Interval	Heights	Number

3. Prepare a list of how the classmates of Mina help their parents most, doing what sort of work in their spare time, as follows:

Name of work	Number of friends
Does shopping	15
Washes clothes	6
Cleans rooms	5
Cooks food and serves	12
Nurses domestic ani-mals	8
Helps in farming	10
Total	…………….

a. Draw a pie chart using the table above
b. Like Mina's class, prepare a list of how the friends of your class help their parents most, doing what sort of work in their spare time and show them in a Pie Chart.

4. The daily wages (in Taka) of 30 labourers in a factory are given:

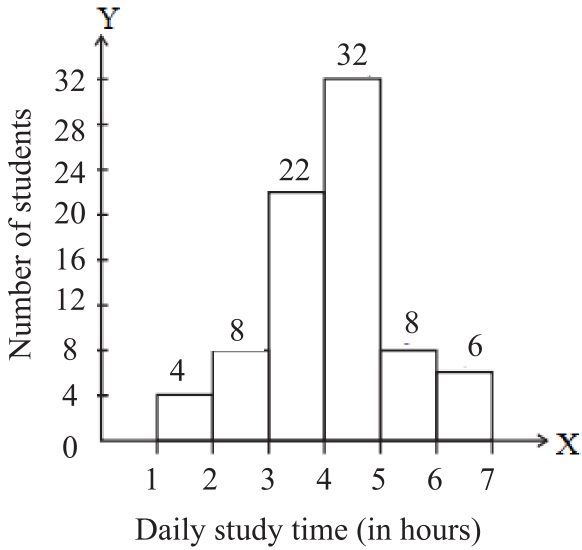
20, 550, 630, 700, 650, 500, 850, 650, 750, 575, 680, 920, 650, 820, 930, 990, 760, 840, 650, 580, 900, 840, 760, 850, 950, 550, 990, 760, 820, 890, 975, 675, 690, 750, 940, 650, 740, 860, 875, 980

a. Determine the range of the data.
b. What are the class intervals of the classes 550 – 599, 600 – 649, 650 – 699?
c. Determine the class number of the data according to the class intervals obtained in 'b'
d. Prepare a frequency table using the tally marks and draw a histogram.
e. Determine from the histogram, how many labourers' wages are above Tk 800.

5. A diagram of the daily study time (in hours) of 80 students is given below. Observe the diagram carefully and answer the following questions:

a. What is the name of the diagram? Write down its characteristics.

b. What is highest time the students spend on study?
c. How many students study less than 4 hours?
d. How many students study more than 5 hours?



Carefully observe the information below, think, discuss with friends, is necessary. Then draw the most appropriate diagram in each case and explain, with reasons,

a. Fill up the Table of the birth months of all the students in your Class and draw a diagram.

Name of month	Tally marks	Frequency
January		
February		

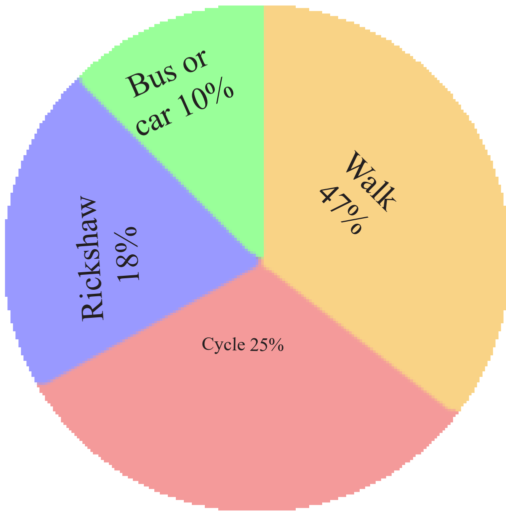
b. The weights (in kg) of the family members of Angel, Shumit, Nipa, and Minoti Costa are as follows:

30.2, 8.5, 11.6, 45, 32.8, 65.3, 38.4, 48.6, 55.5, 26.9, 40.8, 17.6, 22.3, 68.2, 48.5, 56, 62, 36.4, 67.3, 52.8

c. In various sectors of the development planning in one of the district, the percentage of the allocated money is as follows:

Sectors	Farming	Industry	communication	Electricity	education	Others
Allocated Taka (%)	30	25	15	8	12	10

6. Matin found out by asking 720 students, how they travel to school. The following Pie Chart is drawn of the information Matin obtained. Observe the diagram and answer the following questions.



   a. How many students walk to school?
   b. How many students go to school riding bicycle?
   c. Determine the number of students who come by Rickshaw.

7. The Mathematics teacher took a test of 100 marks to verify the proficiency in Mathematics of the students of two sections of class seven. He found out after evaluating the scripts that some students obtained marks less than 20 and some students obtained marks more than 70. So he divided the marks in the intervals 0 – 20, 20 – 30, 30 – 40, ……., 70 – 100 and constructed the following table. Draw a Histogram with the Data in the Table.

Marks	0 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 100
Frequency	8	9	12	16	20	15	20

স্বাধীনতার
৫০
বছর
উন্নয়ন আমারও

১৩ তম এসএ গেমস এ ১৯ টি স্বর্ণ, ৩২ টি রৌপ্য ও ৮৭ টি ব্রোঞ্জ পদক জয় বাংলাদেশের ক্রীড়াবিদদের

দক্ষিণ এশিয়ার মাল্টি ইভেন্ট স্পোর্টস এর সবচেয়ে বড় আসর এসএ গেমস (সাউথ এশিয়ান ফেডারেশন গেমস)। ১৯৮৪ সালে নেপালে এসএ গেমস প্রথমবারের মতো অনুষ্ঠিত হয় এবং দু'বছর পরপর প্রায় নিয়মিতভাবেই দক্ষিণ এশিয়ার বিভিন্ন দেশে অনুষ্ঠিত হয়ে আসছে। বাংলাদেশসহ দক্ষিণ এশিয়ার সাতটি দেশের ক্রীড়াবিদগণ এই গেমসে অংশগ্রহণ করেন। ২০১৯ সালে নেপালের কাঠমান্ডুতে অনুষ্ঠিত হয় ১৩ তম এসএ গেমস। পদক অর্জনের বিবেচনায় এটিই বাংলাদেশের এসএ গেমসে সেরা অর্জন। সঠিক পরিকল্পনা ও অনুশীলনের মাধ্যমে ক্রীড়ায় উত্তরোত্তর সাফল্য অর্জনের দিকে এগিয়ে যাচ্ছে বাংলাদেশ।

101010

2023

Academic year

Mathematics
Class VII

সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন কর

- মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

আলস্য দোষের আকর

তথ্য, সেবা ও সামাজিক সমস্যা প্রতিকারের জন্য '৩৩৩' কলসেন্টারে ফোন করুন

নারী ও শিশু নির্যাতনের ঘটনা ঘটলে প্রতিকার ও প্রতিরোধের জন্য ন্যাশনাল হেল্পলাইন সেন্টারে ১০৯ নম্বর-এ (টোল ফ্রি, ২৪ ঘণ্টা সার্ভিস) ফোন করুন

শিক্ষা নিয়ে গড়ব দেশ শেখ হাসিনার বাংলাদেশ

Ministry of Education